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THE SUSPENSION OF SOLIDS IN FLOWING WATER.

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WITH DISCUSSION.

Considerable space in this paper is devoted to the historical side of the subject because the sources of information are widely scattered, and it is desired to indicate, so far as possible, the origin of the ideas and observations upon sedimentary movements which have become common knowledge. In the second part of the paper a comparison of particular facts and observations leads to certain general conclusions with reference to the manifestation of the phenomena studied. The concluding portion is devoted to an analysis of the different explanations of the cause of suspension, for the purpose of building up a satisfactory theory.

To conduce to uniformity and clearness, the following symbols will be used throughout this discussion :

g = 32.2 ft. per second = acceleration of gravity.
F = area of right section of body considered.
h = head of water corresponding to the velocity v.
v = mean velocity of the stream in the vertical considered.
v_1 = surface velocity at the vertical considered.
v_o = bottom velocity at the vertical considered.
i = inclination of the water surface.
f' = tangent of the angle of sliding friction.
f = tangent of the angle of rolling friction (properly a distance).
γ = heaviness of the liquid considered. For the purposes of this article, $\gamma = 62.5$ lbs. per cubic foot = heaviness of fresh water.
γ' = heaviness of the solid considered.
P = resultant thrust, in the stream direction, exerted by a moving liquid upon a solid.
V = volume.
G = weight.
M = mass.
k = constant determined by experiment.
Z = mean depth of stream.
z = variable depth below surface.
b = width of stream.
r = radius of sphere.
q = liquid discharge per unit width of stream.
d = solid discharge per unit of width.

Part I.—Historical Development of the Problem[1].

The varied phenomena incident to the flow of rivers have demanded the consideration and even the anxiety of riparian owners from a time far antedating the development of the modern science of hydraulics. Wherever rugged slopes discharge their melted snows or heavy rains are gathered from steep, impervious water-sheds, a mountain torrent has its birth, and the householder in the valley early learned to study its varying humors. The rapid descent of the Apennines to the sea

[1] It is proposed here to select, from the mass of literature touching upon this problem, only those discussions and observations which seem to mark a distinct ste toward its final scientific solution.

and the consequent turbulent character of the streams of northern Italy, made this a fruitful field of study to a people whose scientific spirit had already gone far toward establishing the fundamental laws of fluid motion.

During the latter part of the seventeenth century means were sought for the amelioration of these mountain rivers. Dominique Guglielmini, physician and hydraulician, was employed by Venice and other Italian cities to prepare plans looking toward the prevention of their ravages. His greatest work was the building of the levees on the Po above Plaisance, and his writings[1] gave the first impetus to a scientific study of fluvial phenomena.

In 1773 Johann Silberschlag produced his comprehensive treatise on hydraulics[2], covering this field to some extent, but it was left to Dubuat[3] in 1786 to publish the first experimental studies which can be considered authoritative. These experiments were made at Paris by order of the French government. His determination of the different velocities at which solid particles begin to be moved by flowing water has been accepted by subsequent writers, and with his work begins the scientific knowledge of the movement of alluvions in sedimentary rivers.

Dubuat's artificial canal was formed of planks 12 pieds[4] in length, 3 pouces thick and 18 pouces wide, so fitted that the form could be altered from rectangular to trapezoidal by the addition of supplementary bracing. Its total length was 132 pieds, and in the trapezoidal form it had a clear bottom width of 5¾ pouces to a surface width of 3 pieds. Water was discharged into the canal under a maximum head of 7 to 8 pieds.

Surface velocities were measured with floats, noting the time of traversing 10 toises[4]. For determining bottom velocities a small ball of mastic was first adopted. Its specific gravity was such that it lost $\frac{11}{12}$ of its weight in water, and was thus very easily moved. Later, more satisfactory results were obtained with red currants. These were

[1] "Della natura di fiumi trattato fisico matematico." Guglielmini, Bologna, 1697.

[2] "Hydrotechnik oder des Wasserbaues." Johann Silberschlag, Leipzig, 1773. Copy at Zurich Polytechnikum.

[3] "Traité d'Hydraulique." Dubuat, Paris, 1786. Page 57, Volume I. Third edition published in 1816. Copy at Zurich Polytechnikum and at École des Ponts et Chaussées, Paris.

[4] "Système ancien" of France—

1 pouce = 1.066 ins.
1 pied = 1.066 ft.
1 toise = 6.395 ft.

These units were in use previous to 1812. Between 1812 and 1840 the "système usuel" was in vogue. Its values are slightly larger—1 pouce = 1.093 ins.

smoother, moving with less friction, and could be more easily seen. The time of passing through a distance of 60 pieds was noted.

The bottom velocities at which various materials began to be moved by the current were as follows:

Potter's clay (beginning with a velocity of 45 pouces, it continued to be carried away as the velocity was gradually decreased to 7 pouces. At 7 pouces a deposit of fine sand took place, which continued on down through a velocity of 4 pouces, until at 3 pouces per second the clay ceased to show action)	3 pouces per second.
Gravel (size of anise seed)	4 pouces.
Gravel (size of peas)	7 "
Coarse sand (sand remained stable while bottom velocity was increased from 3 up to 7 pouces. At 8 pouces it began to be entrained and for velocities of 12 to 45 pouces per second it continued to be entrained and suspended)	8 "
Sea pebbles (1 pouce diameter)	24 "

Dubuat's experiments also showed that a current velocity of 10 or 12 pouces per second was sufficient to produce sand waves in a bottom whose grains were large enough to be easily visible. He describes these furrows as perpendicular to the longitudinal axis of the current with a short steep down-stream face and a long gentle posterior slope. Each sand grain was slowly rolled along the up-stream incline and fell of its own weight down the crest, thus advancing the wave by steps equal to the diameter of the grains.[1] He computes a velocity under these circumstances which requires two years to cover a length of 2 400 toises.

The expression now universally used to represent the thrust exerted by a current against a solid of any form was deduced by Dubuat[2] and the coefficients experimentally determined.

He argued that the pressure on the up-stream face (p_1) would be greater and that on the down-stream face (p_2) would be less than the

[1] For a similar description see "Report of Chief of Engineers, U. S. A.," 1875, II, pp. 502-504.
Also "Handbuch der Wasserbaukunst." G. Hagen, 1871, Zweiter Theil, "Die Ströme," S. 161-162.

[2] See Flamant, "Hydraulique," 1891, p. 561.

pressure (p) at the corresponding points if the solid were removed. Therefore, the total impelling force would be

$$P = F\,(p_1 - p_2) = F\,[(p_1 - p) + (p - p_2)].$$

These pressures can be written as functions of the velocity height h, whence

$$\frac{p_1 - p}{\gamma} = m\,\frac{v^2}{2g} \text{ and } \frac{p - p_2}{\gamma} = n\,\frac{v^2}{2g},$$

when m and n are experimentally determined.

By substitution,

$$P = (m + n)\,\gamma\,F\frac{v^2}{2g} \text{ or, as usually written,}$$

$$P = k\,\gamma\,F\cdot\frac{v^2}{2g}.$$

The fact that floating solids move with a velocity superior to that of the current which bears them was noted by Dubuat[1]. His explanation of it was inaccurate, but the phenomenon itself has an important bearing on the present discussion.

Of interest in this connection is the experimenter's statement with reference to the theoretical form of bed best adopted for flowing streams. He rejects the rectangle and semicircle as being unable to sustain their own weight in soft soils and chooses the trapezoidal cross-section as offering a proper talus. These right lines will be rounded by the stream itself, the slope being proportioned to the diminishing velocity from center to sides.[2]

Dubuat's work further includes various studies into the régime of rivers and the development of rules for that radius of curvature at bends which will best conduce to stability. To him belongs the honor of inception along these lines.

J. A. Fabre was the next to publish systematic studies[3] on the movements of solids in torrents and rivers, followed in 1811 by the voluminous encyclopedia of Wiebeking.[4] These men extended the range of observed data without making material additions to the theory of fluvial action.

In the year 1845, Bouniceau[5] discussed at considerable length the

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[1] "Principes d'Hydraulique," Dubuat, No. 220. Quoted by Durand-Claye, *Annales des Ponts et Chaussées*, 1886, 1, 530.

[2] See "Principes d'Hydraulique," Dubuat, 1786, Vol. I, p. 119.

[3] "Essai sur la theórie des torrents et des rivières," J. A. Fabre, Paris, 1797, Première Partie.

[4] "Wasserbau." C. F. Wiebeking, Munich, 1811-1817, 4 volumes.

[5] "Étude sur la navigation des rivières à marées et la conquête de lois et relais de leur embouchure." Bouniceau, Paris, 1845.

shifting of sands in tidal estuaries, and showed himself a close student of the laws of erosion by water action. His excellent little volume brings to light and discusses a number of anomalies in this form of action.

He gives a set of values of the bottom velocities at which erosion begins to take place with different materials :[1]

Clay	0.08 — 0.15	m. per second.
Coarse sand	0.22 — 0.30	"
Coarse gravel	0.11 — 0.61	"
Ordinary pebbles	0.65 — 1.00	"
Stones (size of an egg)	1.00 — 1.20	"
Conglomerates	1.52	"
Sedimentary rock	1.83	"
Solid rock	3.00	"

The River Garonne for a length of 45 miles below the embouchure of the Lot was made the object of a series of observations covering 11 years by M. Baumgarten.[2] These measurements deal with the varying discharges from month to month, with the constant changes in form of cross-section and maximum depths, determinations of the fall and heights of water as well as with geological and meteorological studies of the valley. It has formed the model for later fluviatile studies. Here are given the first measurements of discharge of detritus[3] which the author has been able to find. Daily samples were taken at Marmande from the surface of the river in a vessel containing 4.6 liters. This was allowed to stand for nine or ten days, the clear water decanted and the sediment filtered until thoroughly dry. After weighing, a simple calculation gave the weight in grams of mud per cubic meter of water. These measurements were continued from 1839 to 1846 continuously, and the average monthly solid discharge of the river at this point computed.[4] M. Baumgarten distinguishes three different methods of movement common to these solids.

[1] "Etude sur la Navigation," etc., p. 19.

[2] "Navigation fluviale, Garonne." M. Baumgarten, Ingénieur ordinaire. *Annales des Ponts et Chaussées*, 1848, 2, 1-157.

[3] Same, pp. 47, 146.

[4] In order to see if the water contained the same amount of suspended matter at all depths, Baumgarten made a series of tests of specimens from different depths and taken from points where the velocity was different.

From the results given in the table in the continuation of this note on the opposite page he decided that a surface specimen gave a fair average.

First.—A discontinuous rolling motion along the bed of the stream which takes place when the velocity of the current is limited or the materials large.

Second.—With greater velocities or smaller particles, a discontinuous suspension in the lower laminæ of the current.

Third.—Movement in continuous suspension when the particles are carried throughout the entire length considered.

The sand waves which M. Dubuat had observed on a small scale are reproduced in the Garonne on a large scale in gravel shoals, and M. Baumgarten made careful measurements of the yearly progress of one of these crests.

In 1840 the talus down stream had a vertical height of 1.3 m., a base of 2.8 m., while the length of the crest was 180 m. In 1841, the form was nearly the same, but the crest had moved down stream parallel to itself about 30 m. In 1842 the forward motion was 20 m. These gravels were of about the size of a walnut, and the velocity of the water averaged 2.25 m. per second.

Thus far attention had been especially directed towards the phenomenon of dragging, and the laws it follows had been, to some extent, investigated. Inspecteur-Général Dupuit,[1] in 1848, emphasized the true importance of suspension in the movements of soft river bottoms, and to him is due the first scientific study of the causes which produce this action.

Dates.	Depth at which the water was taken.	Weight of Filtered Sediment.	
		In dead water of a bridge or in a gentle current.	In a strong current.
		Grams.	Grams.
March 25th, 1847..	at the bottom at 7.0 m....	0.72	....
	at 3.5 m..................	0.75	....
	at the surface............	0.82	....
March 27th, 1847..	at the bottom at 8.0 m....	0.34	....
	at 4.0 m..................	0.32	....
	at the surface	0.19	....
April 9th, 1847....	at the bottom at 8.75 m...	1.29	0.93
	at 4.40 m.................	1.43	1.18
	at the surface......	1.13	1.22
April 15th, 1847....	at the bottom at 9.0 m....	1.89	1.90
	at 4.50 m.................	1.78	2.15
	at the surface............	....	1.60
April 18th, 1847....	at the bottom at 8.0 m....	0.94	0.90
	at 4.00 m.................	0.87	0.68
	at the surface............	1.33	0.87

[1] "Etudes théoriques et pratiques sur le mouvement des eaux." Paris, 1848. Second edition, Paris, 1863, pp. 214–229. J. Dupuit, Inspecteur-Général.

Dupuit calls attention to the experiment of revolving rapidly a glass of water containing sand grains. He notes that there is a direct relation between the velocity of the water and the amount of sand in suspension, and that the grains tend to arrange themselves in successive laminæ according to the order of their size; as the velocity is decreased, they descend successively to the lower strata. These facts had all been observed before his time, but Dupuit goes farther than his predecessors in noting that the maximum amount of suspension, *i. e.*, that in the lower layers, corresponds, not to the greatest absolute velocity of the current, but to the maximum relative velocity of contiguous molecules. This is a distinct step in advance. Dupuit finds here his explanation of the phenomenon of suspension.

Starting with the fact first noted by Dubuat that the velocity of a float exceeds that of the current,[1] he calls attention to the tendency of such bodies to move toward the filaments of greatest velocity and explains this upon the principle of least work. Assuming the resistance to its motion to vary with the direction of its path, this direction will necessarily be that which offers the least resistance. Therefore an oblique path toward the most rapid current in the stream line will result, since this will offer the least difference in velocity between the solid and the fluid and so the least frictional work.

Dupuit derives a law for this lateral movement as follows:

Let v = the absolute longitudinal velocity of the body.
u = the absolute transverse velocity of the body.
w = absolute velocity of filament at shore side of body.
w' = absolute velocity of neighboring filament toward center of stream.

The relative velocity of the body as regards the liquid surrounding it may be expressed by:

$$\sqrt{u^2 + v^2} - \frac{w + w'}{2} \quad \text{......(1)}$$

Considering the resultant of the resistances which the body suffers as approximately proportional to this relative velocity, the value of u may be found for which this resultant is a minimum.

Calling Q the angle of inclination of the tangent to the curve of velocities at the point considered,

$$w' = w + u \tan. Q \quad \text{......(2)}$$

[1] Dupuit, in common with Dubuat, ascribes this excess of velocity to the accelerating force represented by the component of the bodies' weight parallel to the surface of the current. M. Du Boys, *Annales des Ponts et Chaussées*, 1886, 1, 199–242, has clearly demonstrated the incorrectness of this explanation.

Substituting (2) in (1) and putting the first differential coefficient of the expression equal to zero, he finds that the resistance will be a minimum for

$$u = \frac{\tan. Q}{2} \sqrt{u^2 + v^2}$$

i. e., the transverse velocity should decrease with the tangent of the curve of surface velocities, or, in other words, from the banks to the center of the stream. This is equivalent to saying that the maximum lateral velocity will correspond to the maximum relative velocity of the filaments.[1]

Applying this same law to the velocities considered in a longitudinal section, he finds a resultant force acting obliquely upward, which produces the phenomenon of suspension. As this force will be greatest where the relative velocities are greatest, *i. e.*, near the bottom, the lower laminæ will carry the heavier load of particles. Solids of equal density will arrange themselves from bed to surface in the order of their volume. Suppose, now, the relation of a solid to neighboring ones is considered. The presence of another will tend to decrease the relative velocities of the filaments, and so the two will be obliged to descend to a lower lamina than would the one alone. Descent or ascent will follow according as the bodies approach each other or separate.

Dupuit formulates these laws as follows:

"*First.*—A water current can suspend solids of a density superior to its own.

"*Second.*—The power of suspension depends upon the relative velocity of the filaments and is greater according as this relative velocity is greater. In general, it is proportional to the quantity $\frac{dv}{dz}$ (where v = velocity of current and z = depth below surface) so that lower layers can carry either more solids or those of greater volume.

"*Third.*—The power of suspension of a bed is limited, *i. e.*, a square meter of cross-section can only carry a certain number of solids of a definite volume. Thus each lamina has a different degree of saturation."

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[1] Observations made by Major Cunningham in the Ganges Canal seemed to indicate a current from the shore to mid-stream whose intensity followed this same law (see *Proceedings* of the Institution of Civil Engineers, Vol. LXXI, p. 66). As this current was indicated only by the behavior of certain floats, it is more in consonance with present knowledge to believe their action due to the cause given here by Dupuit than to suppose an actual lateral motion of the water.

Dupuit assumes a river flowing with section and fall unchanged, and saturated with sediment. The entire load will be carried to the embouchure. Suppose the section to vary. At each change will come a change in the curve of velocities and a consequent change in the power of suspension.

When this power is reduced, there will follow a deposit, and when it is increased, erosion will take place. He makes it clear also that these results are dependent, not only upon the section at the point where the change takes place, but also upon the anterior portion of the river as affecting the state of saturation in which the river reaches the section in question. These effects can be brought about at any point whatever by suitably changing the up-stream section.

When a deposit occurs the material comes wholly from the lower laminæ, and they, in turn, receive from the upper ones the material in excess of their power of suspension. This explains the lamination of river-beds in materials increasing in size with depth below the bottom. The frequent presence of beds of finer particles interrupting this structure he explains by the principle that saturation may be obtained either by the size of the particles or by their nearness together.

The numerous variations to which this laminated movement is subject is noticed, and explains the constant rising and falling of particles from one lamina to another, while the nature of the horizontal curve of velocities is such as to cause a constant movement of particles from the banks toward the center. To this may be attributed the tendency of a river to form islands in the middle of its bed at the expense of its banks.

Among the German writers of this period, the discussion of the transportation of stones by torrents was especially taken up by Joseph von Gumppenberg Pöttmes[1], but no further experiments were published until 1857, when Blackwell[2], in England, extended the investigations of Dubuat to solids of larger dimensions.

The velocities given in the table on the next page are those at which movement began:

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[1] "Der Wasserbau an Gebirgsflüssen," Joseph Freiherrn von Gumppenberg Pöttmes, Augsburg, 1854.

[2] See "Report of the Referees upon the Main Drainage of the Metropolis," July 31st, 1857, Appendix IV. Also, for table here quoted, see *Proceedings* of the Institution of Civil Engineers, Vol. 82, p. 48.

Description of substance.	Cubic Contents.		Velocities.		Increase of contents of substances moved.	Increase of velocities.	Sixth root of increase of contents.
	1. Cubic inches.	2. Cubic inches.	1. Feet per second.	2. Feet per second.			
Brickbats........	2.59	18.5	1.75 to 2.00	2.75 to 3.00	7.14	1.37 to 1.70	1.38
Brickbats........	4.76	18.5	2.25 to 2.50	2.75 to 3.00	3.97	1.10 to 1.33	1.26
Oolites...... ...	2.39	17.68	2.00 to 2.25	2.75 to 3.00	7.40	1.22 to 1.50	1.39
Flints...........	1.95	10.37	2.50 to 2.75	3.00 to 3.25	5.32	1.09 to 1.30	1.32
Slate............	2.38	9.06	2.00 to 2 25	2.75 to 3.00	3.81	1.22 to 1.50	1.25

The important idea of saturation with solid material is definitely stated[1] by M. Scipion Gras in a valuable paper[2] published at this time.

He defines saturation in a stream as that state at which the least addition to the solid material already carried will cause a deposit, and its power of entrainment as the total weight of material which a given stream in a state of saturation can carry. He assumes this power of transport to vary directly with the velocity, density and depth of the water, and, these quantities remaining constant, to vary with the volume, density and form of the solids submitted to its action. Upon these principles he explains erosion as a necessary consequence, when the saturation corresponding to the actual velocity is incomplete, unless the bed offers too great a resistance.

Measurements of the advance of the crests of shoals, similar to those undertaken by Baumgarten in 1840, were made by Hübbe[3] in 1861 on sand bars. His observations show the same wave form on a large scale, which Dubuat had noticed in the minute form, and confirm Baumgarten's statement of the forward motion of the crests.

The results of the exhaustive study of the Mississippi River[4] and

[1] Probably first stated by Frisi, "On Rivers and Torrents," 1732. See Report on Mississippi River. Humphreys & Abbot, pp. 190, 415.

[2] "Études sur le torrents des Alpes." M. Scipion Gras, *Annales des Ponts et Chaussées*, 1857, 2, pp. 1–96.

[3] *Zeitschrift für Bauwesen*, Jahrgang xi, 1861. Abstracted in *Zeitschrift des Architekten und Ingenieur-Vereins*, Hanover, 1863, p. 518.

[4] "Report on the Mississippi River." Humphreys and Abbot, 1861. Reprinted with additions, Washington, 1876.

its delta were published in 1861, and contain a wide range of data on the distribution of sediment. Observations along the same lines as those of Baumgarten were instituted at Carrollton in 1851, and lasted throughout two years. They were conducted by Prof. Forshey. Samples were taken from a point near the east bank, where the high-water depth was 100 ft., from the middle of the river, and from a point near the west bank, where the high-water depths were 100 and 40 ft., respectively. These tests were made daily (except Sundays) and samples taken from surface, mid-depth and bottom by means of a small weighted keg, with valves opening upward, which was designed to allow free passage to the water until it reached the desired depth. At the station near the west bank only surface and bottom samples were taken. An average value was obtained for the weight in grams of sediment to 600 grs. of water at each of the positions, 100 grs. of the water being measured out into its proper precipitating bottle for each of the six working days of the week, and corresponding to each of the eight positions.

During the second year samples were taken only from the surface and at the position near the east bank. The tabulated results of these measurements are given in Humphreys and Abbot's Report (edition of 1876, pp. 134, 417).

From the study of these results, Humphreys and Abbot drew the following conclusions:

"This table is fruitful in results. It establishes that the Mississippi water is not charged to its maximum capacity with sediment, because the distribution of the material is different from what must have place were this the case. Dupuit demonstrates that the power of suspension is due to the fact that the different layers of water are actuated by different velocities, and thus exert different pressures upon the different sides of the suspended atoms. Hence, the greater the difference in the velocity of consecutive layers, the greater will be the power of suspension. Now, it is conclusively proved in Chapter IV[1] that the change of velocity from layer to layer is, in horizontal planes, greatest near the banks and the least near the thread of the current; and in vertical planes, parallel to the current, the greatest near the bottom and surface, and the least at a point about 0.3 of the depth below the surface, where the absolute velocity has its maximum value. If, then, the water be either charged to its maximum capacity or overcharged with sediment, we must find the greatest amount near the banks and

[1] "Report on the Mississippi River." Humphreys and Abbot.

near the surface and bottom, and the least amount near the thread of the current and near the layer 0.3 of the depth below the surface. If the water be undercharged, on the contrary, the distribution of sediment will follow no law, the amount at any point being fixed by the accidental circumstances of whirls, boils, etc., although, of course, there will be an accumulation of material near the bottom, where the suspending power is very much greater than elsewhere. Bearing these well-established principles in mind, an inspection of the preceding table must convince any one that the Mississippi water is undercharged with sediment, even in the low-water stage. A most important practical deduction may be drawn from this fact, namely, the error of the popular idea that a slight artificial retardation of the current, that caused by a crevasse, for instance, must produce a deposit in the channel of the river below it."

Sediment observations were also made at Columbus by the Mississippi survey from March to November, 1858, but, as only surface specimens were taken and no tabulated results give a means of comparison between the amounts in suspension near the banks and at the thread of the current, they can be of little service in the scientific study of the distribution of sediment in the cross-section of the river. Curves are shown, however, on Plates XII and XIII of their Report,[1] from which the relation between the mean velocity of the river and the corresponding mean amount of suspended matter at Carrollton and Columbus may be seen. The values from which these curves are plotted are given at page 417 of the same Report (edition of 1876).

From them Humphreys and Abbot are led to the same conclusion as before, *i. e.*, that the Mississippi water is not saturated with sediment, using the term in the sense in which it is used by M. Scipion Gras.[2]

Their line of reasoning is as follows: If the water be at all times charged with sediment to the maximum capacity allowed by its velocity, then the amount of sediment at different stages must vary proportionately with the mean velocity.

"At the date of highest water, both in 1851 and 1858, the river held in suspension but little more sediment per cubic foot than at dead low water. * * * Moreover, it will be seen that an analysis of the distribution of sedimentary matter held in suspension leads to the same

[1] "Report on the Mississippi River." Humphreys and Abbot.

[2] See page 249.

conclusion, by establishing that the river is never charged to its maximum capacity of suspension."

Extreme care was taken in all these measurements in determining the amount of sediment in the sample obtained. It was shown that determinations of sediment must be made by weight and not by volume, as the latter method introduced discrepancies. These were due to the difference in density of the sediment, resulting from different methods of manipulation by various observers.

An extended series of measurements had been in progress on the Elbe, at Harburg, during the years 1837 to 1855, by Baurath Blohm.[1] The data obtained were minutely examined and formed the nucleus for a work treating of the subject in all its bearings. The early death of Herr Blohm prevented the publication of anything but the introductory part of the proposed book. Reference will be made to these observations later.

In 1871, M. Partiot published studies[2] on the movement of sands in the Loire, and enriched the knowledge of the subject by minute observations and extensive measurements. This monograph deserves especial mention, as it brings out strongly the importance of vortices and eddies in the suspending power of water.

Attention is called to the interaction of the suspended particles in changing their forms by friction, to the suspension and disintegration of the clays in the higher layers and their mixture with vegetable matter to form the rich alluvial deposits which settle on the summit of the shoals at the embouchure of the Loire. The slower moving sands are carried only intermittently in suspension.

Measurements are given to show that the quantities of sediment decrease toward the river mouth, and the interesting point is determined by experiment that the amount of silt varies not only with the height of the flood, with reference to others, but also with its own relative state. Increasing as the flood crest approaches, it reaches a maximum at its summit and descends to a lower point at the middle of the posterior slope than at the corresponding point of the anterior slope.

Partiot emphasizes the idea that the sands are only sustained by

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[1] "Ueber die in fliessenden Wasser suspendirt erhaltenen Sinkstoffe." *Zeitschrift des Architekten und Ingenieur Vereins*, Hanover, 1867, pp. 240–297.

[2] "Mémoire sur les Sables de la Loire." M. Partiot. *Annales des Ponts et Chaussées*, I, 1871.

eddies and vortices. He refers to experiments made at Nantes in 1869. Samples were taken at different depths at a point where the river was straight and free from eddies. Sand was not found in suspension, though when introduced 60 ft. above, in a surface velocity of 1.4 ft. per second, its presence was readily detected. At another point, where there was a marked eddy, grains of sand and mica were seen to surge to the surface and glitter in the sunlight, while grains of quartz were brought up in the receptacle from all depths. When the vortices were rapid, grains could even be taken in the hand.

The production of these vortices and eddies is attributed to the inequalities of the bottom, the solids deposited there, the deflecting action of concave banks and the action of floods. As the flood moves down a river in the form of an attenuated wave, the water flows down the front incline with an accelerated velocity. It overtakes the surface water down stream and flows over it, causing eddies. These grow greater as the crest of the wave is approached, since the fall increases, reach a maximum at the summit and decrease on the posterior portion, where the fall is decreased. This view is corroborated by the corresponding measurements of sediment in suspension.

The great velocities which these vortices reach in time of flood explain the movement of boulders, which could not be taken up by ordinary waters. M. Partiot calls to mind the lifting strength of whirlwinds as a parallel case. An interesting point is brought up in the reference to the action of ice in the movement of these solids. The sand grains and pebbles, as well as large stones, at times become frozen into the ice forming at the bed and banks of streams. With the least rise in the water this may become detached and carried to great distances.

It is to dragging rather than suspension that Partiot attributes the motion of sands in the Loire, and quotes some valuable researches made by M. Sainjon in this connection.

A body immersed in a moving liquid is subjected by the current to a thrust which may be expressed by

$$k \gamma F h = k \gamma F \frac{v^2}{2g}.$$

M. Sainjon takes the constant $k = 1.46$ for a prism, and $k = .60$ for a sphere, or as an average $k = 1$ for the particles making up gravels.

The action of gravity upon this immersed body tending to roll it down stream is put equal to

$$(\gamma' - \gamma)\, V \left(\frac{i}{\sqrt{1+i^2}} - f\right)^1$$

Since i rarely reaches the value $\frac{1}{10}$ it is neglected and the approximate expression becomes

$$- V(\gamma' - \gamma) f,$$

whence the resultant force in the direction of the current becomes

$$P = \gamma\, F\, \frac{v^2}{2g} - V(\gamma' - \gamma) f$$

assuming $k = 1$.

To determine the value of f, M. Sainjon uses the results of the experiments of Dubuat. In this case the bottom velocities were determined at which the various materials ceased to be moved by the current[2], and at this point he considers that the approximate resultant force obtained above may be put = o, *i. e.*,

$$\gamma\, F\, \frac{v^2}{2g} - V(\gamma' - \gamma) f = 0,$$

whence

$$f\, \frac{V}{F}\, (\gamma' - \gamma) = \gamma\, \frac{v^2}{2g},$$

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[1] This value is inexact. The correct value is derived as follows:

Represent the resultant weight in water of the body rolling down the inclined river-bed by $W = V(\gamma' - \gamma)$. Let β = the angle of the inclination of the bed.

The gravity component parallel to the bed = $W \sin.\, \beta$ .... (1)

The normal component of $W = N = W \cos.\, \beta$.

The rolling friction = $F = N \tan.\, Q = W \cos.\, \beta \tan.\, Q$ .... (2)

Therefore the resultant force acting is:

$$W \sin.\, \beta - W \cos.\, \beta \tan.\, Q.$$

or since $\tan.\, \beta = i$ and $\tan.\, Q = f$, this becomes

$$V(\gamma' - \gamma)\left(\frac{\tan.\, \beta}{\sec.\, \beta} - \cos.\, \beta \tan.\, Q\right) = V(\gamma' - \gamma)\left(\frac{\tan.\, \beta}{\sqrt{1 + \tan.^2 \beta}} - \cos.\, \beta \tan.\, Q\right)$$

$$= V(\gamma' - \gamma)\left(\frac{i}{\sqrt{1+i^2}} - f \cos.\, \beta\right).$$

By neglecting i, however, thereby virtually putting $\beta = 0$, this final expression reduces to the same form as M. Sainjon's approximate expression $- V(\gamma' - \gamma) f$ which is the one used. Therefore no results are vitiated.

[2] Sainjon is so quoted by Partiot ("Sables de la Loire," p. 32). In one case, at least, that of large sand, Dubuat states ("Traité d'Hydraulique," Paris, 1786, p. 94) that the velocity given is that at which the sand began to be moved as the bottom velocities were increased gradually from 3 up to 8 pouces per second.

These results are tabulated as follows:

Kind of material.	Bottom velocity of current. Meters per second.	$\frac{v^2}{2g}$	$\gamma' - \gamma$	$\frac{V}{F}$	$\frac{V}{F}(\gamma' - \gamma)$	$\gamma \frac{v}{2g}$	f
Dark potter's clay........	0.081	0.0003	1.64	....	....	......	....
Sand deposited by this clay......	0.162	0.0013	....	... . ..	....	....	....
Coarse sharp yellow sand.	0.216	0.0024	1.36	0.002	0.0027	0.0024	0.88
Seine gravels { Size of anise seed........	0.108	0.0006	1.545	0.001	0.0015	0.0006	0.40
Seine gravels { Size of peas...	0.189	0.0018	1.545	0.003	0.0046	0.0019	0.41
Seine gravels { Size of small beans	0.325	0.0054	1.545	0.0045	0.0069	0.0054	0.78
Rounded sea pebbles of an inch or more diameter......................	0.650	0.0215	1.614	0.018	0.0291	0.0215	0.74
Angular flints (size of a hen's egg)..............	0.975	0.0484	1.250	0.045	0.0562	0.0484	0.86

The mean value of f is 0.68. Eliminating the two values, 0.40 and 0.41, so widely different from the others, the mean would be 0.80. Since 0.68 is approximately the tangent of the slope of a natural talus of ordinary earth, while wet sand and earth should have a greater cohesion, M. Sainjon chooses to use the value $f = 0.80$.

Taking the ratio of the bottom velocity in the Loire to the mean velocity at 0.7 (determined by measurements with a Woltmann's wheel), and assuming in general $\gamma' - \gamma = 1.50$, while $\frac{V}{F}$ equals two-thirds of the diameter for round forms and equals the diameter for angular ones, he computes the following table of velocity limits above which gravels will begin to be dragged.

Size of gravel. Diameter in meters.	Velocity at bottom. Meters per second.	Mean velocity. Meters per second.
0.0025	0.25	0.36
0.01	0.50	0.70
0.04	1.00	1.43
0.10	1.50	2.14
0.17	2.00	2.86
0.38	3.00	4.29
0.67	4.00	5.21

M. C. Lechalas published a memoir,[1] also in 1871, in which he takes exception to the theory attributing suspension to the phenomenon of relative velocities. He urges that this assumes flow in parallel fila-

[1] "Les rivières à fond de sable." *Annales des Ponts et Chaussées*, 1871. Published also, after revision, as an annex to Guillemain's "Navigation Intérieure—Rivières et Canaux." Tome I. Paris, 1885.

ments which corresponds in no wise to movements under great velocities. His explanation attributes suspension to repeated shocks from the molecules of water moving more rapidly than the suspended body and to the action of eddies caused by the banks and bottom. He also calls attention to the fact that the variations in velocity in large rivers are much more rapid in the vertical than in the horizontal direction. To these rapid vertical variations he attributes the formation of some horizontal vortices.

The body of this valuable paper is devoted to an attempt to derive numerical results for the values of the mean depth, mean velocity and fall, in alluvial rivers, which will follow the contraction of its width, throughout a given length, by training walls. Certain parts, however, of M. Lechalas' work have a direct bearing on the relations between velocity and movement of sedimentary matter. It will be seen that he lays stress on the distinction between transportation by dragging and by suspension.

Referring to Dubuat's experiments, he expresses the excess of pressure on the up-stream face of an immersed body as proportional to the square of the velocity of the water surrounding the body, *i. e.*, for sand grains on a river bed,

$$\text{Thrust of the moving water} = a^{1}\, v_o^2,$$

where a is a constant which varies with the dimensions, form and posit.on of the grains of sand.

The resisting force of the sands of the Loire is put equal to $a\ 0.25^2$ since they are not transported until the bottom velocity reaches 0.25 meter.[2]

The resultant force—

$$P = a\,(v_o^2 - 0.25^2)$$

is put equal to the mass of the particle multiplied by its acceleration[3] parallel to the direction of v_o. Measuring the velocity v_o in the direction of the axis of the river, M. Lechalas considers this resultant

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[1] $a\ v_o^2$ corresponds to the expression $\frac{k\ \gamma\ F}{2\ g}\ v^2$ used by M. Partiot, quoted on page 253.

[2] Compare table quoted from Sainjon at page 369. This refers to sands not already compacted by the continued action of currents of velocity too slight to transport the grains, but yet sufficient to increase the resisting power of the surface lamina.

[3] M. Lechalas has used the word *vitesse* here. It must, however, be meant for acceleration.—See "Navigation Intérieure," Guillemain, Annexes—Rivières à fond de sable p. 489.

force proportional to the discharge of sand in the river and puts—

$$a\,(v_o^2 - 0.25^2) = b\,d$$

or

$$d = \frac{a}{b}\,(v_o^2 - 0.25^2) = m\,(v_o^2 - 0.25^2) = m\,(v_o^2 - 0.06)$$

where d represents the discharge of sand per unit of width.

The value of m is to be determined by observation, and in this way a correction made for the use of v_o, the absolute velocity of the water at the bed, instead of the relative velocity of the water and the solid particle. When the velocity v_o becomes greater than a certain value, the particles are lifted and cease to roll on the bottom. The term $-\,m\,0.06$ then disappears and

$$d = m\,v_o^2$$

for particles suspended immediately above the bottom.

The advancement of the crests of the sand bars measured by M. Sainjon in the Loire gives a method of determining the value of d for corresponding values of surface velocity v_1, and also a means of comparing this advancement with the corresponding velocities of the current v_o at the bottom.

TABLE OF OBSERVATIONS ON ADVANCEMENT OF CRESTS OF SAND BARS.

Observed surface velocity in meters per second.	Height in meters of the crest above the down-stream bed.	Displacement of the crest in hundred-thousandths of a meter per second.	
		Observed for a lapse of many days.	Computed.
0.58	0.900	3.0	3.0
0.64	0.300	3.3	3.9
0.73	......	5.1	5.5
0.75	0.782	6.3	5.9
0.81	0.967	6.7	7.1
0.81	......	7.5	7.1
0.83	0.760	7.6	7.5
1.00	0.953	10.5	11.6
1.016	0.920	12.4	12.0
1.016	0.580	12.0	12.0
1.03*	0.487	6.2	12.35
1.05	0.612	7.0	12.9
1.11	1.198	5.8	14.6
1.13	0.650	8.7	15.2
1.33	0.950	5.6	21.6

* In the table quoted above by Lechalas from Sainjon the following note appears: "It is wrong to suppose the co-existence of rolling and suspension for the velocities 1.03 m. * * * * The absolute lack of accord with the law of advancement up to $v_1 = 1.016$ can only correspond to a complete transformation of the method of transport. If there was a mixed period extending between the surface velocities 1.03 and 1.33, the calculated velocities would only differ gradually from the observed ones. Instead of that, we see that beyond $v_1 = 1.03$ the observed movements are not more than half the computed ones. This remaining advancement is explained by the deposit of sands held in suspension—a deposit caused by the sudden diminution of velocity below the crest. No trace of the mixed period remaining when the surface velocity reaches 1.03 m., it is probable that it was about over when the value reached 1.02 m."

The computed values of the displacement of the crest given in the last table are derived from the formula—

$$\text{Displacement} = 0.00013\ (v_1^2 - 0.11),$$

in which M. Sainjon expresses the rate of advancement of the crest so long as the surface velocity does not exceed 1.016 m. per second.

In discussing this formula M. Lechalas calls attention to the fact that this displacement becomes o for $v_1 = \sqrt{0.11}$, which virtually[1] corresponds to a bottom velocity of 0.25 m., and that one could write

$$d = m\ (v_1^2 - 0.11),$$

by giving to m the value 0.00013 × the mean height of the crests in the last table corresponding to the surface velocities from 0.58 m. up to 1.016 m. This mean height is equal to 0.77 m. and the product gives $m = 0.0001$.

Whence

$$d = 0.0001\ (v_1^2 - 0.11) \quad \text{.........} (0)$$

He objects, however, to this form because v_1, the surface velocity, will vary with the depth of the stream and so introduce another variable.

Since the bottom velocity v_o ought not to vary widely, he prefers his own equation

$$d = m\ (v_o^2 - 0.06) \quad \text{.........} (1)$$

to equation (0), preceding, as given by M. Sainjon.

Referring to the preceding table, he adopts the value 1.016 m. as the upper limit for surface velocities at which dragging occurs. Up to this velocity the equation (1) will be used, and beyond this limit the equation

$$d = m\ v_o^2 \quad \text{.........} (2)$$

will be used to express the relation between solid discharge and velocity at the bottom.

It remains to find the value of v_o which corresponds to $v_1 = 1.016$ m. M. Lechalas does this by using the formulas of Darcy and Bazin

$$\frac{Z i}{v^2} = 0.00028 + \frac{0.00035}{Z} \quad \text{.........} (a)$$

$$\text{and } v_1 = v + 14\sqrt{Z i} \quad \text{.........} (b)$$

By combining these two equations,

$$\frac{v_1}{v} = 1 + 14\sqrt{\frac{Z i}{v^2}} = 1 + 14\sqrt{0.00028 + \frac{0.00035}{Z}} \quad \text{......} (c)$$

[1] M. Sainjon (see page 369) considers $v_o = .7\ v_1$; $v_1 = \sqrt{0.11} = 0.331$; $v_o = 0.7 \times 0.331 = 0.23$ m. per second.

Substituting the value $v_1 = 1.016$ m. and the values

$$Z = \text{mean depth} = 0.5 \text{ m.}$$
$$= 1.0 \text{ m.}$$
$$= 2.0 \text{ m.}$$

he derives the corresponding values

$$v = 0.71,\ 0.75 \text{ and } 0.78.$$

By combining Darcy and Bazin's equations

$$v_1 = v + 14\sqrt{Z\,i} \text{ and}$$
$$v_1 = v_o + 24\sqrt{Z\,i}$$

he obtains

$$v_o = v - 10\sqrt{Z\,i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

which combined with (*a*) gives

$$\frac{v_o}{v} = 1 - 10\sqrt{0.00028 + \frac{0.00035}{Z}} \quad \ldots\ldots\ldots\ldots (e)$$

Substituting the values of v which correspond to the assumed values of Z, the values of v_o are obtained—

$$v_o = 0.49,\ 0.56 \text{ and } 0.61.$$

M. Lechalas adopts a mean of these values, $v_o = 0.55$ m., as the upper limit of the bottom velocity corresponding to transport by rolling on the bed of the Loire. Since the range of velocities for which the table on page 257 gives indications of a combined mode of transport occupies such a small part of the velocity scale (from $v_1 = 1.016$ m. to $v_1 < 1.03$ m.), he assumes the same value, $v_o = 0.55$, as the lower limit of the bottom velocity corresponding to transport by suspension.

To show more clearly the actual relation of these velocity limits to the variables of the current, an ideal canal is assumed, of constant width and of a flow equal to 3 cu. m. per unit of width, so that

$$b = 1 \text{ and } q = 3.$$

Equations (*a*) and (*d*) preceding combined with the equation

$$q = Z\,v,$$

which expresses the definition of liquid discharge when b is put equal to 1, give, for $v_o = 0.55$ m.,

$$i = 0.000035,\ Z = 4.50 \text{ m., and } v = 0.67 \text{ m.}$$

and, for $v_o = 0.25$ m.,

$$i = 0.000003,\ Z = 10.00 \text{ m. and } v = 0.30 \text{ m.}$$

To express these results in the words of M. Lechalas:

"A bed of regular width, filled with sand which is not renewed, and which lies at an inclination exceeding a certain limit, receives a

discharge of 3 m. of water per second per unit of width. After a length of time greater or less, according to the fall and the length of the canal, a state of unstable equilibrium establishes itself. The mean depth is then 4.50 m., the mean velocity 0.67 m., the fall 3.5 cm. per kilometer, and the bottom velocity 0.55 m.

"The sand, however, is still transported, but in quantities smaller and smaller each second. After a considerable time a new state of equilibrium is established. This is final; it corresponds to a mean depth of 10 m., a mean velocity of 0.30 m., a fall of 3 mm. per kilometer, and a bottom velocity of 0.25 m. Although these computations apply only to an ideal channel, yet they are of interest as showing what an important role is played by the consideration of these velocity limits in the study of alluvial rivers."

Returning to the equations—

$$d = m\,(v_o^2 - 0.06) \text{ for } 0.25 \text{ m.} < v_o < 0.55 \text{ m.} \ldots\ldots\ldots\ldots (1)$$

and

$$d = m\ v_o^2 \qquad \text{for } v_o > 0.55 \text{ m.} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

M. Lechalas uses the following method to determine the value of m. By combining equations (c) and (e) of pages 258 and 259.

$$\frac{v_1}{v_o} = \frac{1 + 14\sqrt{0.00028 + \dfrac{0.00035}{Z}}}{1 - 10\sqrt{0.00028 + \dfrac{0.00035}{Z}}} \ldots\ldots\ldots\ldots\ldots (f)$$

which gives the ratio between the surface and bottom velocities in the artificial canal used by Darcy and Bazin.

The bottom velocity ought to be less dependent upon the depth than that at the surface. If a formula is expressed in terms of the bottom velocity, it may properly be transformed into terms of the surface velocity and mean depth, or of mean velocity and mean depth. On the other hand, when the formula is in terms of the surface velocity, and it is desired to express it in terms of the bottom velocity, it is necessary to assume the ratio $\frac{v_1}{v_o}$ a constant for all values of the mean depth. This introduces an approximation unavoidable without a new series of observations.

Assuming $Z = 1$ meter[1] in equation (f)

$$\frac{v_1}{v_o} = 1.80.*$$

[1] The mean of the values of Z used to obtain the critical value $v_o = 0.55$ m., and hence the most consistent value to use in determining m.

* For $Z = 3$ m., $\frac{v_1}{v} = 1.60$.

Equation (0) page 258, which is based upon M. Sainjon's empirical formula, may be considered reasonably accurate for the range of velocities for which it is intended, as can be seen from a study of the computed results in the table of page 257.

Equation (0) and equation (1) of page 258 may now be written

$$d = 0.0001\ (v_1^2 - 0.11) = m\ (v_o^2 - 0.06)$$

and, by introducing the approximate value $v_1 = 1.80\ v_o$ from page 260, the value found

$$m = \frac{0.0001\left(\overline{1.8\ v_o}^2 - 0.11\right)}{v_o^2 - 0.06}$$

For $v_o = 0.50$ m. per second.

$$m = 0.00037^{[1]}$$

and M. Lechalas' equation (1) becomes

$$d = 0.00037\ (v_o^2 - 0.06) \quad \text{(3)}$$

and (2) becomes

$$d = 0.00037\ v_o^2 \quad \text{(4)}$$

The objections to the introduction of the uncertain value of the ratio $\frac{v_1}{v_o}$ in obtaining these final equations are all admitted, but M. Lechalas maintains that if a numerical coefficient can be used when the discharge d is expressed in terms of v_1 at the surface,[2] one can be much more reasonably used when the equation is in terms of the bottom velocity v_o.

The years 1874 to 1879 marked the arousal of a great popular interest in the United States in the question of silt movements in the Mississippi. The bitter controversy between the Government engineers and Captain James B. Eads with his associates over the improvement of the mouth of the river need not be entered into here. Suffice it to say that the many spirited articles written on the subject during those years were not of great scientific value and left the knowledge of the distribution of the sediment in the river in the same state of incompleteness in which it was left by the report of Humphreys and Abbot in 1861.

Mr. Eads states his views in a letter[3] of March 15, 1874, with reference to these sediment movements in the following words:

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[1] For values of v_o between 0.40 m. and 0.55 m., the corresponding values of m range between 0.00041 and 0.00036.

[2] As is done in the equation (0) based on M. Sainjon's formula; displacement $= 0.00013 \times (v_1^2 - 0.11)$.

[3] To William Windom, United States Senate, Chairman of Committee on Transportation Routes to the Seaboard.

See "The Mississippi Jetties," p. 28, E. L. Corthell, New York, 1881.

"By far the greatest portion is, however, transported in suspension. The amount of this matter and the size and weight of the particles which the stream is enabled to hold up and carry forward depend wholly upon the rapidity of the stream, modified, however, by its depth. * * * A certain velocity gives to the stream the ability of holding in suspense a proportionate quantity of solid matter and when it is thus charged it can sustain no more. * * * The fact that a given current will keep in suspension a corresponding quantity of solid matter; that at a less velocity a portion of it will be deposited and taken up again at a greater, is fully recognized in experimental science and has been extensively made use of for analysis of soils. An eminent investigator of this subject, Prof. E. W. Hilgard, of the University of Michigan, now of the University of California, Oakland, Cal., has classified silts according to the different velocities at which they deposit.[1] This independent line of research fully confirms the view herein advanced in explanation of the phenomena presented through the alluvial bed of the Mississippi."

Gen. A. A. Humphreys, Chief of Engineers, expresses his views in a report[2] to the Secretary of War, dated April 15, 1874, in the following words:

"It has been recently stated by a civil engineer,[3] in a pamphlet concerning the improvement of the mouths of the Mississippi River by jetties, that the amount of sedimentary matter carried in suspension by the Mississippi River is in exact proportion to the velocity of its current; and that, as a given velocity of current will keep in suspension a corresponding quantity of solid matter at a less velocity a certain portion of it will be dropped. * * * The first statement is in direct conflict with the results of the long-continued measurements made upon the quantity of earthy matter held in suspension by the Mississippi River at Carrollton, near New Orleans, and at Columbus, 20 miles below the mouth of the Ohio, one of the chief objects of which was to determine this very question, whether any relation existed between the velocity and the quantity of earthy matter held in suspension. These results prove that the greatest velocity does not correspond to the greatest quantity of earthy matter

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[1] *American Journal of Science* III, VI, 337.

"The classified table of Prof. Hilgard gives the relative velocities created in a mechanical contrivance made for test purposes in a laboratory in which coarse sand is dropped at a certain velocity of the machine, which may be represented in nature as a current of about 2.5 ins. per second; the finest sand when the current is 0.3 of an inch per second; the coarsest silt when the velocity is 0.14 of an inch per second; the finest silt when the velocity is 0.02 of an inch per second."

"Report of Chief of Engineers, U. S. Army," 1874, Part I, p. 865.

[2] "Report of Chief of Engineers, U. S. Army," 1874. Part I, p. 863.

[3] James B. Eads.

held in suspension; on the contrary, at the time of the greatest velocity of the current at Carrollton, the river held in suspension but little more sediment per cubic foot than when the velocity was least. When the quantity of earthy matter held in suspension was greatest the velocity was 2 ft. per second less than the greatest velocity, the quantity of earthy matter in the one case being three times as great as in the other. We find at another time, when the velocity was one-half the greatest velocity the quantity of earthy matter held in suspension was double the amount. Again, we find the quantity of earthy matter in suspension the same, the velocity in the one case being 6.75 ft. per second, and in the other, 1.5 ft. per second.

I.—Carrollton, 1851.

Date.	Weight in grains of sediment in 1 cu. ft. of water.	Mean velocity of river in feet, per second.	Remarks.
February 20th	450	6.5	
March 20th	200	6.2	
April 15th	150	5.6	
May (last week of)	100	3.75	
June 20th	650	4.3	
July 10th to 30th	450	4.8	
August 1st to 20th	450	From 4.8 to 3.5	Change in velocity regularly decreasing, while suspended matter remains the same.
September 8th	300	3.0	
October and November	100	1.75	
December	175	1.85	
January 20th, 1852	400	2.75	

II.—Columbus. Twenty Miles below the Mouth of the Ohio, 1858.

Date.	Weight in grains of sediment in 1 cu. ft. of water.	Mean velocity of river in feet per second.	Remarks.
April 1st	300	7.00	
April 10th	300	5.25	
April 25th	450	7.25	
May 1st	300	7.50	
May 10th	300	5.75	
May 22d	160	6.75	
June 16th	330	8.25	
July 16th–17th	650	3.75	
August 2d	350	4.75	
August 9th	250	4.00	
September 2d	600	2.50	
September 9th to 23d	200	2.25	
October (all of)	200 to 100	1.50	Uniform decrease in amount of sediment, the velocity remaining the same.

"The tables (on page 263) illustrating what has just been said, have been prepared from the report on the Mississippi River. The figures given express the conditions not only on the day noted, but on several successive days.

"It is to be remarked that the investigations respecting the sediment in suspension show that the quantity depended on the river from which the volume of discharge was at the time chiefly derived.

"The cross-sections, both at Carrollton and Columbus, remained unchanged during the above observations."

In order to define still more clearly the position of General Humphreys on this question, the following quotation is made from his report of 1875:[1]

"It has been sometimes stated that every velocity of current is capable of carrying in suspension a certain fixed quantity of earthy matter, and that the water of a muddy river is always thus charged with the maximum quantity of earthy matter it can carry. * * * But this assumption as to the carrying power of currents is utterly disproved by long series of exact measurements upon the Mississippi River. * * * These measurements upon the quantity of earthy matter suspended in the Mississippi River show that at no time has the water been so heavily charged with it that the current could not carry it along in suspension to the same extent as it did when the quantity of earthy matter was least; and they further show that the current of the Mississippi River, when most feeble, can carry in suspension the greatest quantity of suspended earthy matter found in it to the same extent that it can carry the least quantity found in it.

"It was undoubtedly the observation of facts similar to these that led to the conclusion, entertained by some, that the suspending power of the current of a river did not depend upon its absolute rate of motion, but upon the difference of velocity between the adjoining fillets of water. There is good reason to conclude that this is one of the causes or sources of the suspending power of a stream.

"This proposition, therefore, respecting certain velocities of current always carrying certain fixed quantities of earthy matter, and always adjusting those quantities according to its own variations of strength is so entirely disproved by facts that it will not be considered again."[2]

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[1] Annual Report of Chief of Engineers, U. S. A.," 1875, Part I., pp. 959–975. Reprinted in Humphreys and Abbot's "Report on the Mississippi River." Edition of 1876. Appendix M, p. 684.

[2] Those readers who wish to go farther into the details of this somewhat amusing controversy are referred to Humphreys and Abbot's Report on the Mississippi River, Edition of 1876. Appendices.

Review of same by James B. Eads, M. Am. Soc. C. E., in *Van Nostrand's Engineering Magazine*, Vol. XIX, 1878, pp. 211–229.

Answer to Mr. Eads' attack by General Henry L. Abbot, *Van Nostrand's Engineering Magazine*, January, 1879, Vol. XX, pp. 1–6.

Answer to General Abbot by Mr. Eads, *Van Nostrand's Engineering Magazine*, Vol. XXI, 1879, p. 154.

An article by Mr. G. K. Gilbert,[1] upon the erosion of the Colorado cañons, appeared in the *American Journal of Science*, July and August, 1876. While subject to some criticism, it may be regarded as a most valuable contribution to the knowledge of the laws of transport of solid bodies by water currents. It is believed that Mr. Gilbert is the only writer who has called attention to the fact that the same expenditure of energy will transport a greater weight of fine particles than of coarse ones of the same density.

A series of observations was conducted by Assistant Engineer J. B. Johnson at Helena, on the Mississippi River, in 1879.[2] Longitudinal and transverse soundings were made to determine the existence and movement of sand waves in the river bed, and the results plotted[3] so as to show clearly the presence of these undulations. From the observations Mr. Johnson deduces the following facts:

"Average length of waves from crest to crest, about 100 m.

"Extreme length of waves from crest to crest, about 150 m.

"Average height of waves from crest to valley, about 5 ft.

"Extreme height of waves from crest to valley, about 8 ft.

"Average velocity of motion of crest, 5.41 m. per day.

"These results were obtained in a depth of water varying from 13 to 30 ft. The stage of the river varied from 12 to 18 ft. above low water at Helena. The waves decreased in size for a falling river and *vice versa*. Their rate of motion down stream is a function of the velocity of the water. They do not extend from bank to bank at Helena but disappear about 200 m. from each shore, covering about 1 000 m. of the cross-section of the river."

Sediment measurements were made by the same party from March 1st to June 18th, 1879, and deserve special mention because of the introduction of an improved sediment can[4] for bringing up specimens from the bottom.

Samples were taken each day from the surface and 1 ft. above the bottom at points one-fourth and three-fourths of the distance across the river. Proportions of sediment were determined by weight in the later experiments and the mean velocity of the river was determined by floats upon five occasions during the extent of the observations.[5]

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[1] See digest in *Engineering News*, August 19th, 1876.

[2] See "Report of Chief of Engineers, United States Army," 1879, Part III, pp. 1963-1970.

[3] See Plate I, p. 1966. "Report of Chief of Engineers," 1879, Part III.

[4] For sketch, see "Report of Chief of Engineers, United States Army," 1879, Part III, p. 1965.

[5] The tabular result of these observations is given at p. 1969 of above Report.

Simultaneous observations of a like nature were conducted at St. Louis[1] by R. E. McMath. They are more satisfactory in that they offer a slight opportunity for study of transverse distribution of sediment. Both sets give velocity measurements.

The most extended set of observations published upon sediment movements and sand waves are those instituted by the Mississippi[2] and Missouri[3] Commissions in 1879-1881. These were made at St. Louis, Carrollton, Prescott, Winona, Clayton, Hannibal, Grafton and St. Charles. They are wide enough to put at rest certain debated questions, but yet fail in several points to be completely satisfactory—notably in failing to give data on horizontal distribution. These measurements will be again referred to.

Major Allan Cunningham made a series of observations on the Ganges Canal[4] to determine the amount of sediment carried and its distribution in the cross-section.

A tube 12 ft. long, open at both ends, was thrust down vertically from a floating boat until the bottom was reached. It was then closed at the bottom, by a lid worked by a spring, and the column of water, extending from bed to surface, carefully separated from its sediment by decantation and filtration.

This sediment, when weighed, gave a result, called silt-density, which represented the average density in the vertical examined.

To determine the distribution of silt, two collections were made, by the method indicated, at each of nine points in the width of the canal at two different cross-sections. Each set was completed as rapidly as possible. The mean silt velocity past each vertical was computed by multiplying this silt density by the corresponding mean velocity. Captain Cunningham then plotted three transverse curves on a common base, using as ordinates the silt density, the mean silt velocity, and the mean velocity past each of the nine verticals. From the want of relative connection between these curves he concludes that in the Ganges Canal there is no close relation between the silt and the velocity at different parts of the channel, and that the silt density at any point varies from instant to instant.

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[1] *Van Nostrand's Engineering Magazine*, 1883, p. 33.

[2] "Report of Chief of Engineers United States Army," 1883, III.

[3] "Report of Chief of Engineers, United States Army," 1887, IV.

[4] "Roorkee Hydraulic Experiments," Roorkee, 1881, Chap. XXIV. Abstracted and discussed in *Proceedings* of the Institute of Civil Engineers, 1882, Vol. LXXI, pp. 1-94. Same reproduced in *Van Nostrand's Engineering Magazine*, April and May, 1883.

In continuation of these measurements, 73 collections were made at four of the cross-sections, the depth and velocity at two of them being very different. These results led to the conclusion that the mean silt density in no way depended upon the depth or velocity in this canal, but rather upon the state of the supply water from the Ganges.

The best known formula for the determination of the size of particles dragged by a current of a given velocity is that proposed by Mr. Wilfred Airy, and derived by him as follows:[1]

Let a = the length of the largest cube the current could move.

Then weight of cube $= \gamma' a^3$ (γ' const.).

Friction of cube on bed of river $= f' \gamma' a^3$ (p const.).

Total pressure of current on exposed face of cube $= k\, a^2\, v_o^2$ (k const.)

For equilibrium—

$$f' \gamma' a^3 = k\, v_o^2\, a^2$$

whence

$$a = \frac{k}{f' \gamma'} v_o^2$$

therefore the weight of the largest cube which a current with a bottom velocity v_o could move would be

$$\gamma' a^3 = \gamma'\left(\frac{k}{f' \gamma'} v_o^2\right)^3 = \frac{k}{f'^3 \gamma'^2} v_o^6$$

If G' and G'' were the weights of cubes of silt, etc., which could just be moved by currents of bottom velocities v_o' and v_o'' respectively, then

$$\frac{G'}{G''} = \left(\frac{v_o'}{v_o''}\right)^6 \quad \text{......................} \quad (1)$$

or, numerically:

If v_o is increased by $\frac{1}{8}$ of itself, it will move particles of twice the weight, since $\left(\frac{1}{1\frac{1}{8}}\right)^6 = \frac{1}{2}$; and if the velocity v_o is doubled, it will move particles of 64 times the weight, since $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$.

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[1] See condensed description in *Proceedings* of the Institution of Civil Engineers, Vol. 82, p. 25. Notation changed.

See Church's "Mechanics of Engineering," p. 831.

A formula, showing that the scouring power of a natural stream is proportional to the seventh power of the velocity, is said to have been proposed about 1855, by W. Hopkins, of Cambridge, England.

See Baldwin Latham in *Proceedings* of the Institution of Civil Engineers, Vol. 82, p. 43.

Mr. Henry Law shows this formula to be also applicable to the case of a cube rolled along instead of sliding, and to be true for a sphere as well as a cube.[1] His proof follows:

The moment of resistance of the cube to turning about its edge is

$$\gamma' a^3 . = \frac{a}{2} = \gamma' \frac{a^4}{2}$$

The turning moment of the thrust of the current is

$$k\, a^2 v_o^2 \;.\; \frac{a}{2} = \frac{k\, a^3 v_o^2}{2}$$

At the instant of turning the equation of equilibrium gives

$$\gamma' \frac{a^4}{2} = \frac{k\, a^3 v_o^2}{2}$$

whence

$$a = \frac{k}{\gamma'} v_o^2$$

Following the same process used by Airy above, this leads to his formula (1).

In the case of spheres, assume each one to be resting upon three others.

Weight of sphere $= \frac{4}{3} \pi r^3 \gamma'$.

Let r sin. $\beta =$ lever arm of weight about point of turning.

Then moment of resistance to turning is

$$\frac{4}{3} \pi r^3 \gamma' . r \sin. \beta = \frac{4}{3} \pi \gamma' r^4 \sin. \beta$$

Thrust of the current $= k \pi r^2 v_o^2$.

Its lever arm about the point of turning would be r cos. β.

Then the turning moment due to the thrust would be

$$k \pi r^3 v_o^2 \cos. \beta.$$

For impending motion, the equation of equilibrium gives

$$k \pi r^3 v_o^2 \cos. \beta = \frac{4}{3} \pi \gamma' r^4 \sin. \beta$$

whence

$$r = \frac{3}{4} \frac{k}{\gamma'} v_o^2 \cot. \beta.*$$

This again leads to Mr. Airy's equation (1) as above, β being constant as r varies.

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[1] See *Proceedings* of the Institution of Civil Engineers, Vol. 82, pp. 29-30. Notation changed.

* Through an oversight, Mr. Shaw has obtained an incorrect numerical coefficient for this last equation in having used, for the value of the section of the sphere normal to the current, $\frac{1}{2} \pi d^2$ instead of $\frac{1}{4} \pi d^2$. It is corrected here.

Mr. Shaw then concludes that the weight of particles moved by a current, whether cubes or spheres, and whether the action be sliding or rolling, will vary as the sixth power of the mean velocity of the current impinging on them, if cohesion between the particles be disregarded.

M. J. Thoulet, Professor of Science, at Nancy, published in 1884 the results[1] of some experiments made to determine the force required to keep particles of different sizes and densities suspended in water.

The apparatus used consisted of a glass tube placed in a vertical position and connected at its lower end by a rubber tube with a stopcock to regulate the velocity of a water-current ascending through the glass tube. The water was led away by a waste-pipe connected near the top, and the velocity for each experiment determined from the weight of water flowing. The details of the experiments were carried out with scientific exactitude.[2]

M. Thoulet computed the mean velocity of the current required in tubes of four different diameters (2.2, 4.775, 6.75 and 8.0 mm.) to hold unmoved, at a fixed point, spheres of different sizes and densities. These spheres were lead bullets of different calibers and balls of wax containing, in their interior, grains of tin, lead or copper. Their sphericity was tested under the microscope, and in all cases they were kept at the specified height for a length of time not less than 30 seconds.

From his results M. Thoulet has computed a table giving in millimeters per second the velocities of vertical currents of water capable of holding in suspension, at a fixed height in the tube, spherical grains of known radii and of given densities. These radii vary from 1 to 2.5 mm. and the densities from 1.5 to 4. The table[3] also gives, in milligrams, the thrust of the current against the grain.

This thrust is equal to the resultant weight of the grain immersed in water, *i. e.*,

$$\tfrac{4}{3}\,\pi\,r^3\,(\gamma'-1) = \text{thrust}$$

and M. Thoulet has computed the values corresponding to the different values of r and γ' from this formula.

Making the assumption of spherical grains in a stream bed, he considers that each one may be regarded as resting on three others, and

[1] *Annales des Mines*, 1884, I, pp. 507–530. For digest, see *Annales des Ponts et Chaussées*, 1885, I, pp 492–500.

[2] See description in *Annales des Mines*, 1884, I, pp. 507–530.

[3] The same, p. 521. See p. 312 of this paper.

shows graphically that for a horizontal movement a force will be required sufficient to move the grain up a slope of about 37°.[1]

This force $= \frac{4}{3} \pi r^3 (\gamma' - 1) \sin. 37°$.

By referring to his table M. Thoulet determines the bottom velocities required to exert a force equal to that demanded by this formula for the three cases given below.

Material.	Diameter of grains in millimeters.	Velocity required in millimeters per second.
Coarse mud	0.40	40.00
Fine sand	0.70	59.68
River sand	1.70	109.58

M. Vauthier, in a valuable paper[2] before the French Association for the Advancement of Science, in 1884, developed mathematical expressions for the velocity, at any instant, of a solid body falling through a liquid, and for the path described in a given time.

His method consists in writing the accelerating force equal to the mass multiplied by the acceleration of the body, assumed to be a sphere.

Accelerating force = weight of body — resisting force

$$= \frac{4}{3} \pi r^3 (\gamma' - 1) - \pi r^2 k \frac{v^2}{2g}$$

When the motion has become uniform the accelerating force will be zero, and one may write for this case.

$$\frac{4}{3} \pi r^3 (\gamma' - 1) - \pi r^2 k \frac{v'^2}{2g} = 0 \quad \ldots\ldots\ldots\ldots \quad (1)$$

whence,

$$v' = 2 \sqrt{\frac{2}{3} \frac{r\, g}{k} (\gamma' - 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where v' represents the limiting velocity, after which motion is uniform.

For any stage of the motion

$$\frac{4}{3} \pi r^3 (\gamma' - 1) - \pi r^2 k \frac{v^2}{2g} = \frac{4}{3} \frac{\pi r^3}{g} \gamma' \frac{dv}{dt} \ldots\ldots \quad (3)$$

$$= \text{mass} \times \text{acceleration.}$$

[1] This is the value of the angle β in Mr. Shaw's analysis preceding.

[2] "De l'entrainement et du transport, par les eaux courantes, des vases, sables et graviers."

L. L. Vauthier, "Mémoires de l' association française pour l' avancement des sciences," Blois, September 8, 1884.

Abstracted at length in the "Mémoires de la Société des Ingénieurs Civils de France," 1885, 2. pp. 29-35.

General results also given in *Engineering News*, November 1, 1884, p. 211.

Subtracting (1) from (3) and simplifying

$$k\,(v'^2 - v^2) = \frac{8}{3}\,r\,\gamma'\,\frac{d\,v}{d\,t}.$$

Separating the two variables

$$\frac{3}{8}\,\frac{k}{r\,\gamma'}\,d\,t = \frac{d\,v}{v'^2 - v^2} \qquad (4)$$

Since $v = o$ for $t = o$ one may write:

$$\int_o^t \frac{3}{8}\,\frac{k}{r\,\gamma'}\,d\,t = \int_o^v \frac{d\,v}{v'^2 - v^2}$$

Integrating by partial fractions

$$\frac{3}{8}\,\frac{k}{r\,\gamma'}\,t = \frac{1}{2\,v'}\left(\int_o^v \frac{d\,v}{v' + v} - \int_o^v \frac{-\,d\,v}{v' - v}\right)$$

$$= \frac{1}{2\,v'}\,\log._e\left(\frac{v' + v}{v' - v}\right)$$

whence,

$$e^{\frac{3}{4}\,\frac{k}{r\,\gamma'}\,v'\,t} = \frac{v' + v}{v' - v}.$$

Putting

$$N = \frac{3}{4}\,\frac{k}{r\,\gamma'}\,v' \qquad (5)$$

and transforming

$$v = v'\,\frac{e^{Nt} - 1}{e^{Nt} + 1} \qquad (6)$$

and, since $d\,s = v\,d\,t$

$$d\,s = v'\,\frac{e^{Nt} - 1}{e^{Nt} + 1}\,d\,t \qquad (7)$$

whence, by integration, since when $s = o,\ t = o$:

$$s = v'\left[t - \frac{2}{N}\log._e 2 + \frac{2}{N}\log._e\left(1 + \frac{1}{e^{Nt}}\right)\right]^1 \qquad (8)$$

---

[1] Put $e^{Nt} - 1 = u$ and $e^{Nt} + 1 = u + 2$.

Then $e^{Nt} = u + 1$. Differentiating, $du = e^{Nt}\,Ndt$,

or
$$dt = \frac{du}{N\,e^{Nt}} = \frac{1}{N}\,\frac{du}{u+1}.$$

Substituting these values in (7) above

$$ds = v^1\,\frac{u}{u+2}\,\frac{1}{N}\,\frac{du}{u+1} \qquad (a)$$

Separating into partial fractions by the method of indeterminate coefficients (cf. Osborne's "Differential and Integral Calculus," p. 189).

$$\frac{u}{(u+1)\,(u+2)} = \frac{2}{u+2} - \frac{1}{u+1}$$

Substituting this value in (a) and integrating

$$\frac{N}{v^1}\,s = 2\int\frac{du}{u+2} - \int\frac{du}{u+1} + C, \text{ or}$$

$$\frac{N}{v^1}\,s = 2\log._e\,(u+2) - 1\log._e\,(u+1) + C \qquad (b)$$

(*Footnote continued on next page.*)

Assuming $k = 0.5$ and $\gamma' = 2.0$ for a mean of the particles moved in river beds, and $g = 9.8088 = 10$, approximately, M. Vauthier obtains from (2) and (5) (for meter measure).

$$v' = 5.16393 \sqrt{2r} \dots\dots\dots\dots\dots\dots \quad (9)$$

$$N = \frac{1.93648}{\sqrt{2r}} \dots\dots\dots\dots\dots\dots \quad (10)$$

This last equation shows that for particles of slight diameter, the value of N, and consequently of e^N, is very large.

Writing equation (6) in the form

$$v = v' \frac{1 - \frac{1}{e^{Nt}}}{1 + \frac{1}{e^{Nt}}}$$

it is at once seen that the fraction in the second member rapidly approaches the value 1 as the diameter of the particle is decreased, *i. e.*, as N approaches ∞. Therefore v approaches v' asymptotically, and at the limit the two will be equal.

For the same reason given above the transcendental term in equation (8) will be so small as to be negligible for particles of slight diameter.

By numerical substitution in equations (9), (10), and (8). M. Vauthier shows that, for a block as large as 1 m. in diameter, at the end of the first second the velocity will be only about $\frac{3}{4}$ of the velocity limit v', and the transcendental term in (8) will be too large to be neglected, but that, with succeeding seconds, it tends rapidly to approach the value v', while the transcendental term tends rapidly toward o.

When $s = o$, $t = o$, and hence $u = o$, since $u = e^{Nt} - 1$. Substituting these values in (*b*) and solving for C

$$C = -2 \log._e 2.$$

Substituting this value in (*b*) and introducing the values of $(u + 1)$ and $(u + 2)$

$$s = \frac{2\, v^1}{N} \log._e (e^{Nt} + 1) - \frac{v^1}{N} \log._e e^{Nt} - \frac{v^1}{N} 2 \log._e 2 \dots\dots\dots \quad (c)$$

To the second number of (*c*) adding and subtracting $\frac{2\, v^1}{N} \log._e e^{Nt}$,

and collecting similar terms

$$s = v^1 \left[\frac{2}{N} (\log._e (e^{Nt} + 1) - \log._e e^{Nt}) + \left(\frac{2}{N} - \frac{1}{N} \right) \log. e^{Nt} - \frac{2}{N} \log._e 2 \right]$$

Since $\log._e e^{Nt} = Nt$, and

$$\log._e (e^{Nt} + 1) - \log._e e^{Nt} = \log._e \frac{e^{Nt} + 1}{e^{Nt}} = \log._e \left(1 + \frac{1}{e^{Nt}} \right)$$

there results finally

$$s = v^1 \left[t - \frac{2}{N} \log._e 2 + \frac{2}{N} \log._e \left(1 + \frac{1}{e^{Nt}} \right) \right]$$

Compare Rühlmann's "Hydromechanik," p. 699, for a similar solution.

To determine the length of the period of time required before the velocity of particles of different sizes becomes practically uniform, M. Vauthier has expanded equation (6) by division into

$$v = v' \left[1 - \frac{2}{e^{Nt}} + \frac{2}{e^{2Nt}} - \frac{2}{e^{3Nt}} + \dots\dots\dots\dots \right]$$

For all but very large bodies all but the first two terms may be neglected. To find the length of time before the actual velocity will differ from the velocity limit by $\frac{1}{1000}$, one may put

$$\frac{2}{e^{Nt}} = \frac{1}{1000},$$

where $e^{Nt} = 2\,000$, and

$$t = \frac{\log.\ 2\,000}{N \log.\ e} \dots\dots\dots\dots\dots\dots\dots\dots \quad (11)$$

From equations (5) and (11) M. Vauthier has prepared a table for particles of different diameters, showing the length of the path described in the first four seconds, and the length of time elapsing before the actual velocity lacks only $\frac{1}{1000}$ of the velocity limit v'.

This table shows how rapidly the velocity of fall approaches the velocity limit in each case, especially with the smaller particles. It is not until the diameter of the particle becomes as great as 1 m. that there is an appreciable difference between the two at the end of the third second, and even then the difference is slight.

Assuming a particle free to descend with a vertical velocity v in a current of water whose mean horizontal velocity is u, its vertical velocity, except in the case of very large bodies, may be put equal to the corresponding velocity limit v'.

The direction of its path, while not a straight line because of the relative velocities of the filaments of water, will yet, in general, form an angle with the horizon whose tangent is $\frac{v'}{u}$.

If the height above the bottom at starting was z it will reach the bed of the stream after a time

$$t = \frac{z}{v'}$$

and at a distance down stream

$$l = z\frac{v'}{u}.$$

With numerical values M. Vauthier obtains the following results:

Diameter of particle $2r$ (in meters).	Velocity of current u (in meters).	Original height above bed z (in meters).	Time in sinking to bottom t (in seconds).	Distance traversed down stream l (in meters).
0.0001 (mud)	1.00	1.00	19.38	19.38
0.001 (sand)	1.00	1.00	6.12	6.12
0.01 (gravel)	1.00	1.00	1.94	1.94

Suppose this body, falling through the water with a vertical velocity v', meets an upward accidental current with a vertical component equal to v'. It would be kept in suspension so long as the current endured. This is held by M. Vauthier to explain the phenomenon of suspension.

He draws the following conclusions from his study:

"(*a.*) Water does not possess a special property by virtue of which it holds in suspension minute particles of a density superior to its own.

"(*b.*) These particles always move toward the bottom with a velocity which depends upon their density and which is inversely as the square root of their transversal dimensions.

"(*c.*) From the value of these velocities, for materials of a density similar to those which form the surfaces of the beds of water courses, the effects of displacement and of transport observed in streams and rivers is very well explained by the single fact of accidental or permanent currents which act upon the bottom."

In his "Hydraulique"[1] M. Flamant has brought together the most valuable parts of the theories advanced by Dupuit, Vauthier, Partiot, Sainjon and Lechalas. His work is of especial interest in that he calls attention to the bearing upon this question of an article by M. Du Boys[2] intended to complete Dupuit's explanation of the increased velocity of a surface float over that of the mean of the surrounding filaments of water. M. Du Boys has completed the explanation of Dupuit by adding that, while the displaced water and the floating body are alike subjected to the accelerating force due to gravity, yet the resistances to which they are subjected are different. In the displaced water, a portion of the gravity work is lost from the non-parallelism of the filaments and the consequent internal frictions, while in the floating body all the accelerating force due to gravity is used in overcoming the friction on the sides and in producing the increased velocity.

[1] "Mécanique Appliquée, Hydraulique" pp. 290-311. M. A. Flamant, Paris, 1891. Baudry & Cie.

[2] *Annales des Ponts et Chaussées*, 1886, I, p. 199.

In summing up the results of his study, M. Flamant expresses the belief that the power of suspension increases with the quantity $\frac{d\,v}{d\,z}$, with the mean or bottom velocity and with the depth.

Experiments reported by Mr. G. F. Deacon in connection with studies for the Manchester Ship Canal give an accurate description of the detailed action of flowing water upon a bed of sand. His summary of results[1] will be reproduced here.

"The observations were made in a long flat-bottomed trough with glass sides by means of which the behavior of the sand could be accurately observed. The sand was from the estuary of the Mersey, the quantities moved were weighed and the surface velocities of the water carefully measured. When water flowed with a steadily increasing velocity over a surface of such sand, fine pieces of broken shell were first moved, and the surface velocity required to produce such movements was considerably less than 1 ft. per second. At such velocities, however, the sand proper was perfectly stable, and however long the flow continued it remained undisturbed; but the fine pieces of shells at the surface of the sand moved in spasmodic leaps, accumulating wherever the velocity was somewhat less.

"The first movement of sand began at a surface velocity of 1.3 ft. per second. This movement was confined to the smaller isolated grains; and if the same velocity was maintained, the grains so moved ranged themselves in parallel bands perpendicular to the direction of the current, each band taking the form of the well-known sand ripples of the sea shore or sand-bottomed stream, with its flat slope upwards and its steep slope downwards in the direction of the current. At this velocity the profile of each sand ripple had a very slow motion of translation, caused by particles running up the flatter slope and toppling over the crest. The steep downward slope was, therefore, being constantly advanced at the expense of the denudation of the less steep upward slope. At a surface velocity of 1.5 ft. per second the sand ripples were very perfect and traveled with the stream at a speed of about $\frac{1}{2160}$ of the surface velocity. At a surface velocity of 1.75, the ratio was reduced to about $\frac{1}{1050}$, and at a surface velocity of 2 ft. to $\frac{1}{480}$. A critical velocity was reached when the surface of the water moved at 2.125 ft. per second, when the sand ripples became very irregular, indicating greatly increased unsteadiness of motion of the water. Up to this point the whole amount of scour was represented by the volume of the sand waves multiplied by an exceedingly low velocity, always less than the $\frac{1}{480}$ part of the surface velocity of the water. At about this critical velocity of 2.1 ft. per second, the particles rolled by the water up the flat slope, instead of toppling over the steep

[1] *Proceedings* of the Institution of Civil Engineers, 1894, Vol. 118, pp. 93–95.

slope, were occasionally carried by the water direct to the next crest; and as the velocity of the water was gradually increased, an increasing bombardment of each crest by the crest behind it took place.

"At about 2.5 ft. per second, another critical velocity was reached and many of the little projectiles cleared the top of the first or even of the second crest ahead of that from which they were fired. At surface velocities of 2.6 to 2.8 ft. per second, the sand ripples became more and more ghost-like, until, at 2.9 ft. per second, they were wholly merged in particles of sand rushing along with the water in suspension. After this the scour was of a totally different character; the sand and water became mixed, and a constant process of lifting, carrying and depositing of individual particles ensued, the sand being stirred to a depth and lifted to a height dependent upon the velocity."

Mr. Deacon refers to the theory that the weight of sand moved is proportional to the sixth power of the velocity of the water and believes the method of determination of this law to be fallacious. His observations showed that, within the limits of the experiments, the weight of material transported was proportional to the fifth power of the surface velocity or possibly a little more. Two curves are given expressing the results. One shows the relation between the surface velocity and the solid discharge in pounds of sand; the other, the ratio between the surface velocity of the current and the velocity of translation of the crests of the sand ripples.

M. Gallois has described[1] a method of experiment,[2] which throws light upon the problem of suspension. A glass bottle 3 ins. in diameter is used and its flat bottom covered to a depth of 0.2 in. with clean sand. By corking so as to exclude all air and rotating rapidly by means of a twisted cord or a turn-table, the sand is thrown by centrifugal force against the sides of the bottle. The motion of the bottle is communicated to the water progressively from the sides to the center, the sand remaining at the outside. If the bottle be suddenly stopped when the velocity of the water has come to equal its own, the sand will at once project itself from the sides to the center in a cloud, gradually subsiding to form a cone at the bottom, with a vertical axis, whose length increases with the velocity of rotation.

This cone flattens with decrease in velocity, until in still water it assumes the corresponding slope of equilibrium for sand. M. Gallois

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[1] *Le Genie Civil.* See *Engineering News,* March 23, 1893, for a brief digest.

[2] Suggested by Dupuit in 1848. See page 360, and also "Etudes sur le Mouvement des Eaux," Dupuit, pp. 216-217; Flamant "Hydraulique," 1891, p. 302. Footnote.

explains this phenomenon as follows: When the rotation of the bottle is stopped, the water continues to revolve, but is gradually brought to rest by the friction from the sides of the bottle. Since this retarding force communicates its action progressively from the outside inward, the interior filaments soon attain a relative velocity with reference to the outer ones which increases toward the axis of rotation. The sand is pushed toward the center with a force which is proportional to the velocity of the fluid. Consequently the cone flattens as the velocity of rotation decreases.

M. Fargue has recently described[1] some experiments of a similar nature started by him in 1872 and repeated lately at Rouen and Langon. The apparatus used consisted of a circular disc upon which a zinc annular ring, about 0.30 m. high, was fixed. The internal and external radii were respectively 0.50 m. and 1.0 m. The disc was so mounted upon a vertical axis as to be given any desired rate of rotation.

By partly filling the ring with water and carefully increasing the rate of rotation a paraboloid of revolution was soon formed by the water surface.

If a uniform bed of sand and gravel was placed on the bottom before rotation began, and a number of floats at the surface, certain phenomena were seen to occur.

Up to a velocity of 1 rotation in 4 seconds the solids remained unmoved on the bottom. When the time of revolution had decreased to 3.5 seconds, isolated grains of sand and gravel moved to the concave side. This radial movement increased with the speed until, at a velocity of 1 turn in 2½ seconds, the entire mass of gravel was collected on the concave side and showed a somewhat regular surface. The floats, on the other hand, gradually descended the surface of the paraboloid until certain ones became stranded on the convex side or the bottom.

After the conditions had become fixed, the disc was suddenly stopped. The water continued its motion in a state of agitation corresponding to the angular velocity at the moment of arrest. The hollow formed toward the axis was at once filled and the surface became horizontal. The gravel was carried toward the center, with a rapidity corresponding to the angular velocity at which the disc was stopped.

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[1] "Expériences relatives à l'Action de l'Eau Courante sur un Fond de Sable." Paris M. Fargue, Inspecteur-Général des Ponts et Chaussées, *Annales des Ponts et Chaussées*, March, 1894.

When this velocity was 0.74 (time of rotation, 8.5 seconds), the materials covered the bottom almost uniformly. The disc was only bare for a discontinuous strip at the outer edge. When the velocity of stoppage was 1.11 (time of rotation, $5\frac{2}{3}$ seconds), the sand and fine gravel moved rapidly to the convex wall and the average gravel spread itself almost uniformly, except that only a few of the large particles remained at the concave wall. When it was 3.14 (time, 2 seconds), all the gravel was violently thrown toward the center and the fine sands followed in spirals of varying lengths. There was little regularity in the motion of the floats, though they generally kept to the concave bank.

Part II.—Discussion of Observed Data.

The extent of the erosive action of water courses marks it as the greatest factor in that definite movement of the materials of the earth's surface from the high toward the low latitudes, which the modern "Doctrine of Isostacy" has sought to explain by a reverse movement underneath and a subsequent elevation[1].

The study of the torrents of Switzerland and Italy suffices to show the size of individual blocks which may be moved along the bed of a stream[2] or even carried freely in suspension. The burden of detritus brought down in the middle and side moraines of the Unteraargletscher in the Bernese Oberland is spread over a wide area by the headwaters of the Aar River, forming a waste of heavy boulders and coarse gravel covering $\frac{1}{2}$ square mile. Through this wilderness of stone, the milky waters of the river find a tortuous path, carrying in suspension to the Lake of Brienz below the particles of powdered rock ground from the sides of the valley by the daily motion[3] of the glacier. It is the presence of this so-called "gletschermilch," which gives to the Swiss lakes a part of their peculiar and beautiful coloring.

These short but destructive torrents divide themselves naturally into a "sammelgebiet or erosionsgebiet," where the water and solid material is gathered, the "gebiet des murgangs"[4] forming a canal

[1] Compare "Theory of the Earth's Rotation and its Interior Heat," pp. 26–32, Elon Huntington. Rochester, N. Y., 1895.

[2] For a graphic description of the descent of material in a mountain torrent, see Lechalas, "Hydraulique Fluviale," Annexes, pp. 424–428.

[3] The Unteraargletscher has a velocity down the valley of 0.50 m. per day. It was here that Agassiz made his glacier measurements. The Rhône Glacier, separated from the valley of the Aar only by the Nägelisgrätli divide, has a daily velocity of 1.0 m.

[4] For examples of dangerous "murgänge," read Riedel's "Ueber Geschiebe Führung und Murgänge der Wildbäche." *Zeitschrift des Oesterrich-Ingen- und Arch.- Vereins*, 1871, pp. 113 and 151.

through which the semi-fluid mass passes at considerable velocity and with little deposit, the "ablagerungsgebiet," where the solid material is deposited in the main valley, forming a clearly defined cone, with its apex at the point where the torrent issues from the mountain. Lastly, the "ablauf"[1] or bed through which the water, relieved of the mass of its burden, finds its way to the main water course.

A photograph, taken during the summer of 1895, shows clearly these lines of demarcation in the two torrents close above Guttannen on the west side of the lower Haslithal. The axis of each is approximately at right angles to the Aar, into which they discharge. At the foot of this same valley on the eastern slope, above Brienz, lies the small Swiss village of Neuschwanden. At its edge, through an abrupt chasm in the mountain side, and so close as to render the danger to the village an imminent one, issues the cone of dejection of the Lammbach, probably the most destructive in Switzerland. The huge mass of stone and boulders covers a fan-shaped area of approximately a square mile and is largely devoid of vegetation. The slope is nearly uniform from the apex to the banks of the Aar, which it has forced against the further side of the Haslithal. An approximate measurement, made by the author in August, 1895, showed this slope to be about 8 degrees. At that time the side toward Neuschwanden was overlaid with the fresh "murgang" or lava-like mass of gravel and boulders of the preceding autumn which had formed a semicircular cordon about the village and was only deflected from the houses by heavy guide walls. The upper surface is nearly plane and the stream does not, as might be expected, thin out gradually to the edges. It forms a bed of nearly uniform thickness, forking into various divisions at the apex of the cone, while each edge is sharp and clearly defined, marking an abrupt descent to the bottom. In general one may liken the form of these streams to that of those beds of broken stone carefully arranged in prismoidal form one sees in American cities.

That something of an analogous nature takes place in all larger water courses is certain. The difference is one of degree and not of kind. The variance of opinion among authorities now hinges on the ratio between the total amount so moving, in a given river, and the amount carried in intermittent or permanent suspension.

[1] The French writers use only the first three divisions and the corresponding terms "bassin de réception," "canal d'ecoulement" and "cône de déjection."—See Surell "Étude sur les Torrents des Hautes-Alpes." Paris, 1870-72, Dunod.

Piles driven up stream from a caisson of the St. Charles Bridge over the Missouri are said to have been found under the caisson when it reached bed rock.[1] Jas. B. Eads describes the sand on the bed of the Mississippi at the St. Louis Bridge as moving for at least 3 ft. in depth, with a velocity decreasing below the surface.[2] A pile embedded upright in the sand has been seen to move bodily down stream.[3] The velocity of movement in this sense has been determined by means of stakes driven in the bed of the Loire.[4] Other recorded cases are numerous and need not be multiplied here.

The transportation of coarse gravel in free suspension is but another order of the same phenomenon. It has been observed in the Garonne, when dikes have been broken through, and gravel, borne in the upper laminæ of the current, has been carried over the breach and deposited in the fields beyond.[5] In a similar case, masses of gravel were carried over a dike below Pittsburgh[6] and deposited down stream, filling up hollows which had previously existed there.

The law of decrease in mean velocity from the rise to the embouchure of rivers is closely followed by the steady decrease in size of the particles forming its bed and strewn along its banks. The ratio between the amount of solid matter entrained and that of the liquid at any point in a stream has been called by M. Fargue its torrential coefficient. This should decrease from source to mouth with the fall and the mean velocity. M. E. Charlon[7] has made use of the law in the deduction of a formula, by means of which he computes the velocity of a stream from the size of the materials transported by it. The question of corresponding decrease in amount of suspended matter, per cubic foot of water, is a disputed one. M. Fargue[8] holds the view that rivers become more and more muddy toward their embouchures, due to the accumulated transformation of the coarse into fine materials by friction. M. Partiot,[9] on the contrary, states that 300 measure-

1 "The Mississippi as a Silt Bearer." R. E. McMath. *Van Nostrand's Engineering Magazine*, Vol. XX, 1879, p. 227.

2 "Report of Chief Engineer of St. Louis Bridge." J. B. Eads, June, 1868, p. 21. Quoted by R. E. McMath in above paper.

3 *Engineering News*, Feb. 9, 1884, p. 65.

4 See Partiot. "Les Sables de la Loire," p. 43.

5 The same. p. 23.

6 "Report of Chief of Engineers, United States Army," 1876. II, p. 5.

7 See *Le Génie Civil*, Vol. XVII, 1890, p. 170. Note giving formula in *Proceedings* of the Institution of Civil Engineers, Vol. 102, p. 350.

8 "Etude sur la Largeur du Lit Moyen de la Garonne," pp. 12, 13. M. Fargue. *Annales des Ponts et Chaussées*, October, 1882.

9 "Mémoire sur les Sables de la Loire." M. Partiot, p. 21.

ments in the Loire, during the floods of 1856, showed the turbidity to decrease regularly toward the sea. Measurements continued throughout the flood showed, proceeding down stream, the weights of sediment per cubic meter of water to be—

At Feurs ....300 grs.	At Nevers ...210 grs.	At Tours.....212 grs.
Roanne...242 "	Gien .....223 "	Saumur...177 "
Digouin ..191 "	Orléans ..237 "	Nantes....150 "

M. Partiot explains the anomalies shown between Nevers and Orléans by the entry, between those points, of tributaries heavily charged with silt.

A comparison of various measurements in the Mississippi was undertaken by the author. The results would seem to bear out M. Partiot's view.

The available data are the Carrollton and Columbus measurements of 1851 and 1858, the measurements of 1879, at Helena and at St. Louis.[1] The two former were taken during floods. The two latter at a medium stage. The Columbus measurements represent only surface specimens. The Carrollton samples were taken with a defective apparatus. These facts render it impossible to draw any conclusions from a comparison of the 1851 and 1858 results with those of 1879, which seem more reliable.

However that may be, a mean of the results at Columbus and at Carrollton, from the second week of March to the second week of November of their respective years (1851, 1858), shows the proportion of sediment to water, by weight, to be—

Columbus	.000749
Carrollton	.000601

an evident decrease in sediment at the lower station.[2]

There is an apparent co-ordination between the two 1879 measurements which gives more weight to the results obtained. Taking the mean only of the top and bottom measurements, as no mid-depth quantities were taken at Helena, and covering the period from April

[1] See a valuable article by R. E. McMath, in *Van Nostrand's Engineering Magazine*, Vol. 28, 1883, p. 33.

[2] During these measurements the mean velocity of the river ranged at Carrollton from 6 to 1.7 ft. per second, and at Columbus from 8 to 1.5 ft. per second, being, as a whole, considerably higher at Columbus.

10th to June 18th, 1879, at Helena, and April 14th to June 25th, at St. Louis, the proportions of sediment to water, by weight, are—

St. Louis	.002046
Helena	.001079

a much greater decrease toward the river mouth.[1]

The law cannot be demonstrated from the measurements now available. That the effect of interaction among the solids moved on the bed should show a cumulative effect in the increased number of fine suspended particles down stream is to be expected, and is shown in Nature by the increased fineness of the deposits. That it should manifest itself in an increased weight of suspended matter per cubic foot, as held by M. Fargue,[2] does not appear to be substantiated by the limited number of observations at hand.

The question is: Will a stream moving at a given velocity sustain a greater weight of fine particles per cubic unit of water than of large ones of the same density? If so, then a heavier load per cubic foot may be carried at the embouchure of rivers with the same expenditure of energy than in their higher reaches.

Mr. G. K. Gilbert[3] has shown that the same consumption of energy will hold in suspension a greater load of fine than of coarse material of like density.

This may be shown as follows: Assume a stretch in the lower course of a river bounded by the cross-sections A and B. Assume the kinetic energy at the two points to be the same, so that the whole gravity work done by the weight of the stream in its descent is used up in external and internal frictional resistance. Suppose an inch cube of stone introduced at the surface at A. The total energy of the stream has now been increased. The cube reaches the bottom at B. It can only act on the bed between the two points by pressure, but as the friction on the bottom is independent of fluid pressure, this friction is not increased. If the cube sinks at the same rate it would have chosen in quiescent water, it makes no demand upon the energy of the

[1] At Helena, the mean velocity ranged from 4.26 to 3.23 ft. per second, while at St. Louis it varied from 7.21 to 4.0 ft. per second, averaging considerably higher than at Helena.

[2] M. Fargue says ("La Largeur du Lit Moyen de la Garonne," p. 13): "Il s'opère donc, de l'amont vers d'aval, une transformation dans la qualité et dans le mode de transport du débit solide: le débit en gros matériaux traînés sur le fond, qui n'a lieu que sous l'influence de vitesses notables, va en diminuant; celui des matériaux ténus, en suspension dans l'eau et obéissant aux faibles vitesses, va au contraire en augmentant. * * * les eaux deviennent en effet de plus en plus vaseuses à mesure qu'on se rapproche de la mer."

[3] *American Journal of Science*, July and August, 1876, Part II; also abstract in *Engineering News*, August 19th, 1876.

stream. If it sinks more slowly, the difference between the distance actually sunk and the distance which would have been covered in quiet water during the time of transit from A to B, multiplied by the weight of the cube in water, measures the demand upon the stream's energy.

Suppose the same stone to be pulverized into small cubes and again introduced at A. The weight in water is unchanged. The draft upon the stream's energy between A and B is computed in the same way as before and found to be less, because the difference between the distance sunk in quiet water and in the actual case is less. The reason why the small particles sink more slowly is because the collective area at right angles to the motion is greater, and so requires a smaller value of the velocity of sinking in order to keep the total resistance to motion,

$$P = k \gamma F \frac{v^2}{2g}$$

a constant in the two cases. This is required because the work done by the cube in sinking to the bottom must be the same as that done by its component parts in covering the same distance.

It is shown, then, that a less consumption of the stream's energy between A and B is required to suspend the same weight of small particles than where the grains are of the same density and larger size. It follows that the same expenditure of energy will suspend a greater weight of the small particles per cubic foot of water.

This may offer an explanation of the phenomenon observed at Columbus and advanced by General Abbot[1] to prove that no relation exists between velocity and weight of sediment per cubic foot of water. He states that the Ohio and Missouri Rivers move side by side at Columbus in the bed of the Mississippi without mingling their waters, and that, while their velocity is common, the Ohio water has only three-fourths as much sediment per cubic unit as the Missouri water.

If they are actuated by the same velocity it may be assumed that there is the same amount of energy per cubic foot in each case diverted to the suspension of sediment. The sediment of the Missouri is much more comminuted than that of the Ohio, as shown by the appearance of the two streams. An excess in weight of sediment per cubic foot is to be expected, then, in the Missouri water, as the measurements showed, even though the velocity is the same in both.

[1] *Van Nostrand's Engineering Magazine*, Vol. XX, 1879, p. 3.

The problem now becomes one dependent upon the circumstances of each case. If the energy available for the work of suspension per cubic unit of water is the same at the mouth as at the head waters of a stream, the weight of sediment per cubic unit will be greater. This excess will diminish and finally become negative as the ratio of the available energy at the mouth and head waters becomes less.

In general, it would seem that the available energy should decrease rapidly toward the embouchure and be accompanied by some slight decrease in the weight of sediment carried per cubic foot of water.

1. Minor Agents Influencing Sedimentation.

Temperature.—Chemical precipitation, in general, takes place more easily at higher temperatures. The same law appears to obtain in the case of matter in mechanical suspension. The author's attention was first directed to the question by Mr. Allen Hazen, of Boston, who had noticed an appreciable increase in deposition of suspended matter at higher temperatures in the sewage at the Lawrence, Mass., Experiment Station. Bouniceau[1] and Partiot[2] are agreed that river deposits are greater in summer than in winter. The sediment observations of Prof. Riddell[3], and those of Prof. Forshey[4], at Carrollton, on Mississippi water both give corresponding temperatures. The former lasted from May 21st to August 13th, the temperature gradually rising from 72° to 84° Fahr. The corresponding amounts of suspended matter show an irregular but still perceptible decrease. They follow, however, much more closely the fluctuations in the river surface above low water, so that the element of decreased depth and consequent decrease in velocity of flow enters in as a more potent factor in producing the same result.

The sediment curves at both Columbus and Carrollton[5] seem to show an increase in suspended matter during the summer months of June, July, August and September, the temperature of the river water at Carrollton reaching a maximum of 86° Fahr. in August and descending very regularly to a minimum of 39° in February.

1 "Etude sur la navigation des rivières à marées." M. Bouniceau, 1845. Quoted in *Proceedings* of the Institution of Civil Engineers, Vol. 66, p. 5.

2 "Mémoire sur les Sables de la Loire," p. 22; M. Partiot. *Annales des Ponts et Chaussées*, I, 1871.

3 "Report to American Association of Geologists and Naturalists, 1846. Quoted by Humphreys and Abbot, "Report on Mississippi River," p. 142.

4 "Report on Mississippi River," Humphreys and Abbot, pp. 134, 148.

5 The same, Plates XII and XIII.

It is evident that river observations are little fitted for the study of this question because of the complexity of the elements involved in fluvial motion. A simple laboratory experiment on the length of time acquired for mechanical precipitation in quiescent water under different temperatures would determine the matter. The yearly range in temperature in rivers is not great, and its influence on sedimentation will be very limited. In the case of flow in sewers it may assume more of practical importance.

A large number of different measurements in the Elbe[1], made under various conditions, failed to show any change in temperature with depth. In the Mississippi River the difference between surface and bottom temperatures is usually too small to be registered by an ordinary thermometer. The maximum difference is a small fraction of a degree.[2]

Herr Blohm[1] has assigned an important place among the causes of suspension of finer particles to a system of circulation set up by the differences in temperature. He calls attention to the fact that water reaches its greatest density at 3.5° R. (39° Fahr.) and that laminæ at a less temperature than this would tend to rise to the surface, as well as those at higher temperatures. The result would be a mixture tending to produce the uniformity actually observed at all depths. The tendency of the warmer laminæ to rise would be equal, in his opinion, to the tendency of the finer particles of sediment to sink in obedience to gravity.

The Mississippi may be taken as an index of the rivers of the temperate zones. Its waters at Carrollton during two years' observations never exceeded the temperature of maximum density, so that, in this case at least, there could have been no circulation, from this cause, of colder water from below to the surface as an equalizer of temperatures.

Light.—The slight molecular agitation caused by the penetration of light has been shown to be sufficient to affect the rate of sedimentation in quiescent water. Mr. Andrew Brown[3] found that a phial of turbid water had a uniform tendency to deposit its sediment most

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[1] "Ueber die in fliessenden Wasser suspendirt enthaltenen Sinkstoffe," Blohm. *Zeitschrift des Architekten und Ingenieur-Vereins*, Hanover, 1867, pp. 277–278.

[2] Lieutenant Marr in "Report on Mississippi River," Humphreys and Abbot, 1876, p. 149.

[3] *Proceedings* American Association for the Advancement of Science, 1848; also, Humphreys and Abbot's "Report on the Mississippi," 1876, p. 144.

rapidly in the portions protected from the light, the surface of the deposit showing a corresponding inclination.

Viscosity.—At low water, under a hot sun, M. Partiot[1] has seen the rising tide, at the embouchure of the Loire, float off patches of sand from the bars and carry them upon its surface so long as it remained undisturbed by waves. A similar phenomenon is noted in the *American Journal of Science*, December, 1890.[2] Blotches of sand, 1 in. in diameter at first, which later joined themselves into 6-in. squares, were eroded from a bank forming an angle of 150° with the water surface. These were seen floating on the surface half a mile down stream, and, if disturbed, would rapidly sink to the bottom. An oiled needle will float on the surface of water if carefully placed in position. Phenomena of this nature are due to what may be called superficial viscosity, which has a greater intensity than the viscosity in the interior of the fluid.

That this latter influence has a part in the suspension of sediment is shown by the length of time required for quiescent turbid water to clear itself. Experiments showed turbidity in water taken from the Garonne after eight days and muddy water from the Elbe made no perceptible deposit until after a lapse of 24 hours.[3] In water taken from the Mississippi at St. Louis in 1865, Mr. Flad[4] found that for a total of 1 000 parts in suspension at the beginning of the experiment,

944.50	parts	had	settled	during	the	first 24	hours.
22.35	"	"	"	"	"	second 24	"
2.92	"	"	"	"	"	" 48	"
30.23	"	were still[5]	in	suspension		after 96	"

Changes in viscosity and consequent suspending power are a probable concomitant of changes in temperature.

Salt Water.—Observations made in 1839 by Sidell[6] at the mouths of the Mississippi showed that the river water alone required from 10 to 14 days to settle. The admixture of salt in any form reduced the time of settling to between 14 and 18 hours. Mr. Gould[7] found that a few

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1 "Sables de la Loire," p. 36.
2 Noted in *Engineering Record*, December 27, 1890, p. 65.
3 Blohm in *Zeitschrift des Architekten- und Ingenieur- Vereins*, Hanover, 1867, p. 245.
4 "Silt Movement by the Mississippi," R. E. McMath. *Van Nostrand's Engineering Magazine*, 1883, p. 33.
5 Since the present article went to press the author's attention has been called to an article by Mr. Carl Barus (Bulletin No. 36 of the U. S. Geological Survey), in which he shows that a degree of comminution can be secured such that deposition will never take place.
6 "Report on the Mississippi River." 1876. Appendix A, p. 500.
7 "Report of Chief of Engineers, United States Army," 1875, II, p. 36.

pinches of salt thrown into a tumbler containing muddy water from the bottom of the Savannah River caused a much more rapid deposit. Mr. Fargue[1] has found that the same amount of mud introduced into a glass of fresh water and into a glass of salt water shows a difference in the period of settling. The salt water is clear after six hours of repose. To attain the same result requires eighteen hours in the fresh water.

This property of saline solutions has an important bearing on the formation of bars at mouths of rivers discharging into salt seas. The load of detritus will be dropped sooner than if the receiving body were fresh water.

Action of Waves.—The formation of bars in deep water at the mouths of tidal estuaries has in late years come to be attributed to wave action upon the detritus discharged by the river rather than to the simple process of deposition itself. It is the dynamic effect of the waves which heaps up the bars.[2] All sea beaches show this action so clearly that there can be little doubt as to its influence, in a lesser degree, on the movement of detritus in rivers. Observation has shown that sands are often moved when the bottom velocity is such as to be an insufficient cause. Mr. P. O'Meara[3] has observed this motion by diving to the bed of a tidal channel where the bottom velocity was too slight, unaided, to move the sands. He found that the sand at and near the bottom, under a depth of 10 ft., had an oscillatory motion corresponding to the 6 and 8-in. waves passing above. At the center of the wave passage the sand reached a considerable velocity; at its end the motion ceased and even seemed to be reversed. He holds that this action may be perceptible to depths of 40 or 50 ft. Waves of translation stir the water to an infinite depth, theoretically, their velocity of translation being dependent upon the depth. Such waves will be confined to tidal estuaries. Waves of oscillation, such as the wind ripples in rivers, are felt, however, to considerable depths.

At Cherbourg[4] these waves cease to act on the piers at a depth of

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1 "Étude sur la Largeur du Lit Moyen de la Garonne." *Annales des Ponts et Chaussées*, Oct., 1882, p. 21 (footnote).

2 A formula for the scouring power of waves, giving relation between height of wave and size of particle moved, is developed by Mr. W. Smith in *Proceedings* of the Institution of Civil Engineers, Vol. 100, p. 201.

3 *Proceedings* of the Institution of Civil Engineers, Vol. 118, pp. 84, 85. Noted, also, in *Engineering Record*, Feb. 23, 1895, p. 219.

4 "Les Marées Fluviales," M. Comoy, 1881, p. 23.

about 25 ft. At Algiers the limit is 35 ft. There the sands cease to be moved at depths between 50 and 100 ft., while the limit for muds is 450 ft. A visit to the harbor of Algiers during a heavy blow showed its peculiarly exposed position so that these figures may be reasonably considered maxima.

Action of Ice.—The removal to great distances of boulders which neither ordinary nor flood velocities could move has been attributed to the transporting power of ice. There is a tendency in rivers to form what is called anchor ice at the bed and sides when the water is shallow. This ice attaches itself to the solids in its vicinity, and, because of its slight specific gravity, is easily detached by flood velocities and carried with its load down stream. M. Partiot[1] calls attention to this action on sand shoals barely covered by water from which the surface layer is detached by the floating away of the ice.

Action of Sediment in Diminishing Velocity.—Mr. Baldwin Latham reports[2] observations covering a series of years which seem to show that the velocity of turbid water for the same depth and fall is less than that of clear water. He holds that this difference bears a ratio to the amount of turbidity, and is caused by the work used in transporting the material. The discharge of clear water multiplied by its velocity corresponded closely to the combined weight of sediment and water in the corresponding case, multiplied by its mean velocity. Mr. G. K. Gilbert has reached the same conclusion.[3] He states that the total energy of a clear stream is used up in friction on its bottom; that this friction is directly proportional to its velocity. When detritus is carried a certain amount of the energy of the stream is used to keep it in suspension, and this takes place at the expense of friction and consequently of velocity. It is to be remembered, however, that the total energy of the stream has, in the meantime, been increased by the addition of the energy represented by the vertical fall of the solid particles.

The law of the conservation of energy will not admit of any other

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[1] "Les Sables de la Loire," p. 36.

[2] *Proceedings* of the Institute of Civil Engineers, Vol. 71, p. 46.

[3] "The Colorado Plateau Province as a Field for Geological Study," *American Journal of Science*, July and August, 1876. Part II. Abstracted in *Engineering News*, August 19th, 1876.

decision in this matter, though the statement has sometimes been made that such a retardation of velocity does not exist[1].

Mr. Gilbert's statement that the work done by a clear stream is entirely used up in friction on the bed is somewhat at variance with the attitude of the best science of the present day.[2] The energy consumed by intermolecular resistances caused by the complex motion in the interior of the liquid is much greater than that actually used at the earth and air profiles. It should be added, however, that these intricate movements are induced by the bed's rugosities. In general, it may be said, that the total energy is used in friction, through which it is transformed into heat energy.

Assume a portion of a clear stream between the sections A and B. Suppose no difference of kinetic energy between the two stations, then the total energy of the stream expended is used in work done on friction. Introduce a mass of sediment in suspension at A and a demand

[1] See "Silt Movement by the Mississippi," R. E. McMath. *Van Nostrand's Engineering Magazine*, 1883, p. 36. Mr. McMath says: "We have seen that transportation of silt (up to the point of impaired fluidity) is not at the expense of the stream's motion. The work of erosion and suspension is done by the stream, whose velocity must be diminished compared with flow under a like head in a smooth channel, but if the now-yielding bed should suddenly become rigid, the same, or even greater force would be expended upon the obstructing roughness. Therefore though suspension consumes a part of the stream's force the velocity is not necessarily lessened beyond what it would be in the only alternative condition that can be considered, a rigid bed equally rough." This line of reasoning would seem to hold, so far as the actual work done upon the bed of the river between any two points is concerned. The work which would have been expended upon a rigid bed equally rough is now in part expended upon the mobile bed in the same way as before, while the residue is free to be used in carrying into suspension whatever is eroded from the bed between the sections considered. But this theory fails to take account of those external forces of Nature which are continually wearing away cliffs, disintegrating hillsides and introducing at the surface of the stream a mass of debris to be carried, for which the stream's own mechanical action is not accountable. Gravity acts as an external force where banks cave in and throw upon the stream's energy an additional burden. The burden already in suspension at the entrance to the stretch considered must be carried in addition to that considered by this theory. It is to this additional burden that a consumption of energy and consequent retardation of velocity may be attributed.

[2] See Boussinesq "Theorie des Eaux Courantes," Paris, 1872, introductory chapter. M. Boussinesq has shown that neither the friction, rightly called, upon the bed nor the added internal friction due to relative velocities of parallel filaments following stream lines is sufficient to explain the transformation of the energy of the stream, in its descent, into heat energy. He shows that if the velocity at the walls were assumed to be zero so as to attribute the whole work to friction between parallel filaments, the coefficient of interior friction is so small that the central filament, in a semicircular conduit of 1 m. radius and a fall of 1 in 10 000, would acquire a velocity of 187 m. per second before equilibrium was established between the accelerating force and the fluid resistances. It is, then, to the vortices that must be attributed the largest share in this transformation. They largely increase the total interior friction.

See "Journal de M. Lionville," t. XIII, 1868. Also "Theorie des Eaux Courantes," Boussinesq, pp. 2–6.

Compare, also, Prof. Unwin in "Encyclopedia Britannica," article on Hydromechanics.

is made on the stream's energy to keep it suspended to B. The thought at once suggests itself that the total energy has, in the meantime, been increased. In answer, it may be said that the addition has also increased the friction at the bed since the formula

$$P = k\,F\,\gamma\,\frac{v^2}{2g},$$

shows this friction to be a function of the heaviness of the fluid, which in its new compound state has been increased. These two changes tend to counteract each other and to leave still an increased demand upon the energy originally used in the passage from A to B. In consequence, there will result a retardation of velocity at B accompanied by an increase of depth if the supply be constant. For particles of a uniform size and density, this decrease in velocity will increase with the weight of the load per cubic unit of water. The decrease will be less for a given weight of fine particles than for the same weight of large ones, other conditions remaining the same.

The presence of silt, then, retards velocity in two ways:

First.—It uses an amount of the stream's energy in suspending it.

Second.—It increases the heaviness of the composite fluid and so increases friction.

2. Influence of Depth on Transporting Power.

It was once believed by hydraulicians that the adhesion between a liquid and its bed was stronger than the internal cohesion of the fluid itself. It seemed the natural deduction from the decrease in velocity observed near the banks and bed. The hypothesis then took form that the particles next the banks remained stationary, while the main current flowed by in a fluid bed of its own consistency.[1] The experiments of Darcy on deteriorated pipes showed that velocity was a function of roughness, and the incorrectness of the former assumption. Even the outermost particles of the fluid substance have a motion relative to the bed. Is this velocity influenced by the depth? Increased depth means increased weight per square unit of bed, and consequently increased pressure, but experiments have shown that not only the coefficient of fluid friction, but also the friction itself, is independent of the pressure.[2]

[1] This has been shown to be true for capillary tubes by M. Duclaux, of Clermont. See *Annales de Chemie et de Physique*, 4e Série, t. XXV, 1872. For flow in streams and large pipes it appears inadmissible.
See also "Theorie des Eaux Courantes." J. Boussinesq, pp. 1-2.

[2] "Encyclopedia Britannica," Article Hydromechanics.

In the case of a homogeneous solid sliding down an inclined plane the coefficient of sliding friction is independent of the normal pressure, but the friction itself

$$P = f\,N \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

is a function of both quantities, and remains unchanged at all velocities. The liquid prism, sliding down an inclined bed, acts according to other laws so far as frictional resistance is concerned. Here again,

$$P = (k\,\gamma)\left(F\frac{v^2}{2\,g}\right)$$

or

$$P = f_1\,N_1 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

an equation of the same form as before, but made up of quantities formed in a different way. In equation (1) N represents the normal component of the body's weight, is proportional to depth when the prism is homogeneous, and is independent of velocity. In (2) N_1 again represents weight, but this weight is directly proportional to a velocity height, and is independent of depth. Increased depth, velocities remaining constant throughout, will have no effect on friction and hence produce no change in scouring action.

The case is sometimes cited, as a substantiation of the view that depth increases transporting power for the same velocity, of the increased difficulty in wading a deep stream. The example is not a good one. A man's foothold is lost sooner in this case than in a shallow ford because of his increased loss of weight rather than from an increase of transporting power, properly called.

The statement has been made[1] that in practice observation shows the scouring power of a shallow stream at a high velocity to be much less than that of a deeper river running at a slower velocity. The explanation offered by Prof. Unwin[2] would seem to account for a portion of this difference. He attributes to the deep stream the advantage that the particles of its bed may be thrown up to a greater height, and, since the velocity of descent again to the bed should be the same in both cases, will be thus carried farther down stream before being deposited.

Flood waters offer great variations of depth which may be used in the determination of comparative amounts of sediment per unit volume.

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[1] See *Proceedings* of the Institution of Civil Engineers. Vol. 82, 1884, p. 31.
Mr. Law's explanation of the phenomenon is based on the incorrect assumption that fluid friction increases with pressure.

[2] "Encyclopedia Britannica," article Hydromechanics.

These weights will vary for the same depth with the relative stage of the flood, and so complications are introduced. The earlier stages of a heavy rainfall wash down the surface particles loosened by weathering. The later portion of the storm finds a more resisting surface. M. Partiot found traces of this fact in the relative weight of sediment at different stages of the same flood. A safe basis of comparison would seem to be the same relative stages of floods of different heights when the same tributaries are discharging high water.

M. Partiot[1] has been able to detect only a slight increase in the sediment per unit volume in the Loire, with the importance of the flood. M. Fargue[2] states that the suspended sediment is very feeble at low water and reaches its maximum intensity at the flood crest.

Major Allan Cunningham[3] bases the statement that no relation exists between depth and silt intensity, upon a series of observations on the Ganges Canal.

A study of the data at hand will throw the most light on the subject. Fig. 1 gives a graphic representation of data bearing upon this point. Further curves might be added from the extensive measurements of the Mississippi Commission[4] made in 1879–81 at Carrollton, Prescott, Winona, Clayton, Hannibal and St. Louis. The Carrollton and Columbus sediment ordinates are taken from the "Report on the Mississippi" by Humphreys and Abbot, page 417—one for each week of the year. The corresponding mean gauge readings were taken from Plates XII and XIII of the same volume. The St. Louis co-ordinates are from Mr. McMath's paper in *Van Nostrand's Engineering Magazine*, 1883, page 33. The Helena co-ordinates are from the "Report of the Chief of Engineers, United States Army," 1879, Part III, page 1968. The Elbe and Maas measurements are taken from the *Zeitschrift des Architekten und Ingenieur Vereins*, Hanover, 1867, pages 290 and 291.

The method of plotting these curves must be distinctly understood. They start from no common origin. They have no quantitative relation to each other. Each represents only the general trend of direction of the number of points from which it is constructed. These are in most cases widely scattered, but their general direction is clearly defined and

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[1] "Sables de la Loire," p. 22, 1871.

[2] "La Largeur du Lit Moyen de la Garonne, p. 14, 1882.

[3] See *Proceedings* of the Institution of Civil Engineers, Vol. 71, 1882, p. 35. "Roorkee Hydraulic Experiments."

[4] "Report of Chief of Engineers, U. S. Army," 1883, III, pp. 2209 and 2266.

is fairly represented by the lines shown. It is seen at once that their direction, in each case, is such as to show an unmistakable increase in sediment per cubic foot with higher stages.[1] Each is independently constructed and all show the same thing.[2] The author can find no other values to contradict them.[3] This seems to show satisfactorily that weight of sediment per cubic foot increases as the river rises and depth increases. That it proves that transporting power in

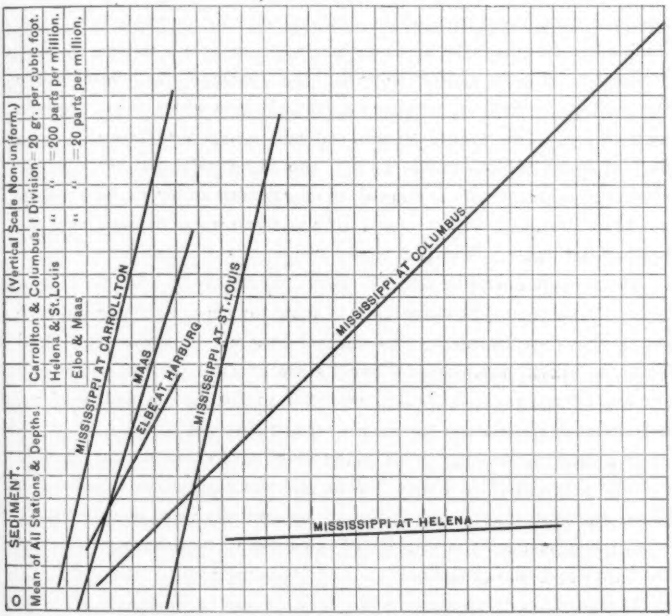

FIG. 1.

creases with depth is not claimed, for velocity increases with stage also, and either one or both together may be the cause of increased

[1] In the curves shown on Plate VIII of the "Report of the Mississippi River Commission," 1879-81 (see "Report of the Chief of Engineers, United States Army," 1883, III), this relation is only slightly traceable in the Carrollton measurements of 1879-80.

[2] Since the above curves were plotted the author has found a plate showing a series of points plotted in an analogous manner from the extensive observations of Assistant Engineer Seddon at St. Charles, Mo., 1879 (see "Report of the Chief of Engineers, United States Army," 1887, Part IV, pp. 3090-96. For description of apparatus used see same report, pp. 3121-23). These points show how a curve similar to those in Fig. 1 could be drawn and are clearly confirmative of the conclusions here deduced.

[3] The results of Prof. Riddell's measurements are confirmatory. See Humphreys and Abbot's "Report on the Mississippi," 1876, p. 142.

transport. If, however, one were to join with Humphreys and Abbot in denying any fixed connection between velocity and suspending power, then this might be looked upon as proof.

The influence of a gradual or sudden decrease in depth upon transporting power offers a range of experiment and observation which is yet to be made. There is little definitely known from measurements about the influence of such shoaling upon the curve of velocities. For unchanged width, decrease in depth means decrease in sectional area and consequent increase in mean velocity for constant discharge. Such a shoaling may be likened, in its action, to a submerged weir. When the change is sudden there will be a mass of dead water above the weir, at the bottom, forming a fluid bed upon which the discharged water flows. This would seem to indicate conditions favorable for a deposit above the obstruction, upon the same principle upon which Humphreys and Abbot explained the formation of delta bars by deposits in the dead angle caused by the fresh water flowing over the heavier salt water.

The case of movable dams in many of the continental rivers is in point. Experience, however, fails to show any shoaling of consequence above these structures.[1] The natural conclusion is that this dead water, so called, must be in a lively state of agitation, its eddies and vortices carrying up deposited material to the higher laminæ whose movement of translation carries it over the obstruction.[2]

In a gradual shoaling, with banks widening to form a pool, there is no sudden accession of vortex motion and the decrease in velocity due to enlargement of section is followed by deposits until equilibrium is established between the velocity of the stream, the resisting power of the banks and that of the bed.

The question of changes in the form of the curve of velocities, as affected by obstructions, is still in need of experimental research by measurements made at varying distances above the obstruction to determine the velocities throughout the range of the back-water.

That the friction of the air has some effect in influencing the form of this curve has been generally accepted since the Mississippi measurements were made. It is only the extent of the influence which was then claimed for this factor which has since been called in question by

[1] For substantiation of this statement see Flamant, *Annales des Ponts et Chaussées*, 1882, quoted by Lechalas, "Hydraulique Fluviale," 1884, p. 64.

[2] See "Report of the Chief of Engineers, U. S. Army," 1876, II, p. 5.

those who have other theories for the cause of the depression of the maximum velocity below the surface.[1] That its retarding effect is less than that of the friction on the bed is usually conceded.

That both are greater than the internal friction due to laminated flow is a reasonable conclusion from the actual form of the vertical curve of velocities, substantiated by Boussinesq's demonstration[2] of the slight value of this friction of laminated flow.

Suppose any cause produces a sudden increase of roughness in the bed and so increased bottom friction. If the hypothesis of parallel filaments[3] is adopted as representing the general trend of flow, the natural conclusion is that the parabolic curve of velocities in a vertical will be tipped down stream. In other words, the lower filaments will be more retarded than the upper ones. That a gradual shoaling with a corresponding increased surface fall has the opposite effect on the curve of velocities has been shown by Dupuit[4] for the case of unchanged width.

There, by a simple numerical calculation, the bottom velocity is shown to take on a much more rapid increase than the surface velocity, so that, at the crest of the shoaling, they have become more nearly equal than before. In the contracted section below the crest the curve will have been tipped up stream, though the rapid increase of bottom friction (varying as the square of the velocity) will soon force it back to the normal position for steady flow at that velocity. He attributes to this disproportionate increase in bottom velocity with the mean velocity, the larger part of the scouring action seen immediately down stream from hydraulic constructions. This view would seem to throw additional light on the cause of non-shoaling above submerged weirs. Experiments are needed on this subject.

3. Influence of Retardation of Velocity.

What, if any, is the relation between velocity and suspension? This is a vital question in the discussion. Assume a sedimentary stream

[1] See Prof. James Thomson, "Encyclopedia Britannica," Article Hydrodynamics. Also, F. P. Stearns, *Transactions* American Society of Civil Engineers, Vol. 12, p. 331, and Vol. 7, pp. 122-130.

[2] See page 289, footnote.

[3] In spite of the statement often seen, that this in no wise corresponds to the complex motion seen in rivers, especially the Mississippi, an ordinary inspection of streams, even in times of flood, shows it to be more reasonable than any other supposition yet advanced, and to represent, in an average sense, the phenomena observed.

[4] "Etudes sur le mouvement des eaux," 2d Edition, 1863, pp. 58-68.

flowing between regular banks. Does any fixed connection exist between the velocity of the mean of all the cubic feet passing a given section per second and the weight of sediment contained in that ideal cubic foot of water? If so, is the relation one of cause and effect? To the first of these questions Humphreys and Abbot give an emphatic negative,[1] based upon a comparison between the Carrollton and Columbus sediment and velocity curves. Captain Ead's criticism of this view was founded on a misconception.

He proves conclusively in his article that the total weight of sediment passing a given point in the river is proportional to the velocity of the current. This, however, was not the question at issue, and is settled by a moment's reflection. The debatable problem is: Does an increase of velocity increase the transporting power per cubic foot of water passing a given cross-section? The vital question in the problem of jetties is not as to the ability of the contracted stream to carry throughout their length and beyond the sediment contained per cubic foot in the water of the river above. Their success hinges upon the capacity of the stream to take an additional load per cubic foot from its increased velocity until the increasing depth has again established equilibrium. The consensus of opinion of writers seems to answer in the affirmative as opposed to the position of Humphreys and Abbot.

Partiot says[2] that sediment in floods follows the same law as the velocities, increasing up to the highest stage and decreasing afterwards.

Referring to the Missouri, Major Ruffner[3] says:

"The water is so heavily charged with sediment that decrease in velocity is immediately followed by a deposit, but the converse of scour following an increase of velocity, although apparent, is not so well marked nor so extensive. * * * When from any cause the velocity of the current is suddenly increased, the most rapid erosion takes place; and the greatest deposit occurs when the velocity is suddenly checked."

Captain Eads and Mr. Corthell[4] state that the current of the Mississippi cannot be checked in the slightest degree in flood time, when its waters are heavily charged, without causing a deposit; that an

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1 See p. 264.

2 "Sables de la Loire," p. 22.

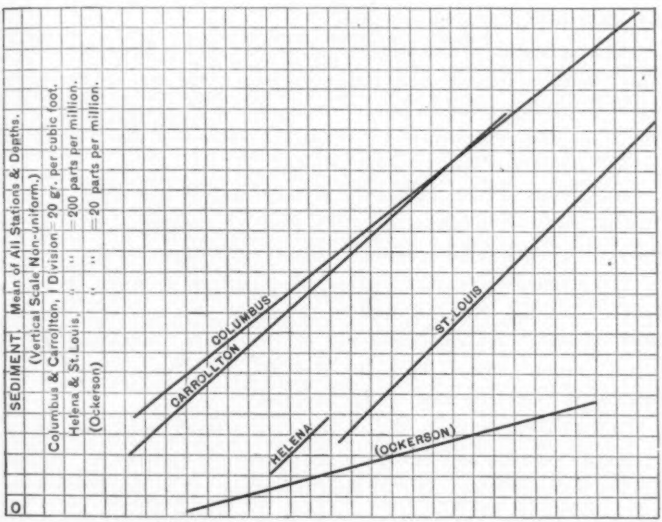
3 "Improvement of Non-Tidal Rivers," 1886, pp. 78, 138.

4 *Transactions* American Society of Civil Engineers, Vol. xi, pp. 262, 263.

iron netting, with meshes 1 ft. square, set in a shoal on the Missouri, caused a deposit of 16 ft. in one flood. Mr. Ockerson[1] claims that the river is not always fully charged with sediment and so may at times be retarded to some extent without deposit.

Major Allan Cunningham[2] states that, in the Ganges canal, measurements have shown that silt density is independent of velocity.

Mr. R. E. McMath[3] says the cause of suspension commonly varies with the velocity.



MEAN VELOCITY. 1 Division = 0.25 ft. per second.

Fig. 2.

These views represent all phases of opinion. General statements are not convincing so it is advisable to analyze the available measurements.

Fig. 2 gives a graphic representation of five different series of measurements in the Mississippi. Other data at hand are unaccompanied by velocity determinations. These curves are plotted with mean velocities as abscissas and amounts of sediment as ordinates. As

[1] *Transactions*, American Society of Civil Engineers, Vol. xi, p. 273.

[2] "Roorkee Hydraulic Experiments." See *Proceedings* of the Institution of Civil Engineers; Vol. 71, p. 35.

[3] "Silt Movement by the Mississippi." *Van Nostrand's Engineering Magazine*, 1883, p. 38.

in Fig. 1 they are not suitable for quantitative comparison. There is no co-ordination in vertical scales. Each represents only the general trend of direction of the points from which it is drawn. The points for the Columbus and Carrollton curves were the most scattering, as would be expected from an examination of the same values plotted in Plates XII and XIII of Humphreys and Abbot's "Report on the Mississippi." There, time is introduced so as to form two curves. Here, a general relation is expressed by one curve, omitting the time element and plotting each observation as a separate point fixed by its sediment and velocity co-ordinates.

An examination of the two curves as plotted by Humphreys and Abbot seems to indicate a relation, especially in the case of Carrollton where samples were taken from three depths and averaged. The relation is not, however, invariable. From the points as plotted here there can be no doubt of the existence of a law.

Another set of measurements was instituted at Carrollton by the Mississippi River Commission in 1879–80 which removes all doubt as to the existence of a relation between velocity and sediment at that point.[1]

The investigations made by this commission at Prescott, Winona, Clayton, Grafton, Hannibal and St. Louis in 1880 and 1881 are the most extensive ever conducted and offer the data for plotting additional curves.[2] Those made by the Missouri Commission at St. Charles[3] in 1879–1880 offer similar facilities.

Of the curves shown in Fig. 2, the most reliable are those representing the Helena and St. Louis measurements because of the improved methods used. In both these cases the law is clearly marked. The curve marked Ockerson is plotted from some measurements published by J. A. Ockerson,[4] M. Am. Soc. C. E.

In Fig. 2 it might seem reasonable to have started each curve from the origin on the ground that stagnant water would carry no sediment. On the other hand, such a proceeding would have been open to objection as an *argumentum in circulo*, and so was avoided. The curves, however, place themselves in a significant arrangement.

[1] See "Report of the Chief of Engineers, United States Army," 1883, III, pp. 2209–2266 and Plate VIII.

[2] These curves have all been plotted by the author and show the same result as those drawn on Fig. 2.

[3] "Report of the Chief of Engineers, United States Army," 1887, IV, pp. 3090–3096.

[4] *Transactions* of the American Society of Civil Engineers, Vol. xi, p. 273. Mr. Ockerson's values for sediment as given above show a marked departure from other determinations in the Mississippi. However, as only the relative quantities are required here, they are used without question.

The curves are believed to show that the sediment by weight in a cubic foot of water does obey a general law of increase with the velocity in the Mississippi River. As such a relation has been largely conceded to exist in other rivers it is believed that it is general. That instant deposit always follow the least retardation is not proved or claimed.

4. Distribution of Sediment in the Cross-Section.

Distribution of Sediment in the Vertical.—Sufficient data is at hand to settle this question in its general bearing. That it has been a matter of dispute is shown by the different opinions expressed.

M. Baumgarten,[1] from measurements in the Garonne, came to the decision that surface measurements were a fair index of the amount of sediment at all depths.

Herr Blohm[2] quotes a number of varying opinions from English, German and Italian engineers; but, from his own measurements in the Elbe at Harburg, finds somewhat of an excess in the surface quantities. He, however, gives it as his opinion that the distribution of sediment is about equal throughout.

Andrew Brown[3] decided, after repeated trials, that sediment in the Mississippi was equally distributed at all depths, provided the samples were taken in the main current.

M. Partiot[4] takes the other view, stating that measurements in the Loire have shown the ratio of surface, middle and bottom quantities to the mean of all, to be represented by the numbers 90, 100 and 110.

M. Surell[5] found that the silt intensity in the Rhone increased rapidly with distance from the surface. He expressed the ratio between surface and bottom amounts by the relation of the numbers 100 and 188.

In the measurements on the Mississippi at Columbus,[6] only surface specimens were used, the ratio 100 to 120 being used to reduce the surface values to the mean for all depths as determined at Carrollton in

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[1] "Navigation Fluviale, Garonne." M. Baumgarten. *Annales des Ponts et Chaussées*, 1842, 2, p. 49. See also page 244 of this paper.

[2] Ueber die in fliessenden Wasser suspendirt enthaltenen Sinkstoffe." Baurath Blohm. *Zeitschrift des Architekten und Ingenieur Vereins*, Hanover, 1867, p. 276.

[3] Humphreys and Abbot's "Report on the Mississippi," 1876, p. 143.

[4] "Sables de la Loire," 1871, p. 24.

[5] M. Guérard on "Mouth of the River Rhone." See *Proceedings* of the Institution of Civil Engineers, Vol. 82, p. 309.

[6] See Humphreys and Abbot's "Report on the Mississippi," 1876, p. 136.

1851–1852, The following table has been prepared from all the data which could be collected upon this matter :

DISTRIBUTION OF SEDIMENT WITH REGARD TO DEPTH.

Place of observation and observer.	Date.	Reference.	PARTS OF SEDIMENT IN 1 000 000 PARTS OF WATER AT THE DEPTHS INDICATED. Surface. (Mean.)	Mid depth. (Mean.)	Bottom (usually 1 ft. above). (Mean.)
Mississippi at St. Louis—McMath	1879	*Van Nostrand Eng. Mag.*, 1883	1 847	2 009	2 117
Mississippi at Helena—Johnson	1879	Rep't Chf. of Eng'rs U. S. A., 1879, III	799	...	1 266
Mississippi at Carrollton—Forshey	1851–52	Rep't on Miss. Humphreys and Abbot, 1861	648	802	842
Sacramento at Kerscheval's—Le Conte	1879	Rep't Chf. of Eng'rs U. S. A., 1879, II	5 525	9 051	5 618
Mississippi at Prescott—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	123	157	159
Mississippi at Winona—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	34	32	36
Mississippi at Clayton—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	40	42	41
Mississippi at Hannibal—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	165	208	224
Mississippi at Grafton—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	319	322	345
Mississippi at St. Louis—Miss. River Com	1880–81	Rep't Chf. of Eng'rs U. S. A., 1883, III	686	906	995
Mississippi at Carrollton—Miss. River Com	1879–80	Rep't Chf of Eng'rs U. S. A., 1883, III	"The means of surface, mid-depth and bottom observations are in the ratio of 100, 144 and 183, respectively." Vide Rep't Chf. Eng'r, 1883, III, p. 2216.		
Missouri at St. Charles—Missouri River Com	1879	Rep't Chf of Eng'rs U. S. A., 1887, IV	2 418	2 473	2 548
Garonne—Baumgarten	1847	Annales des Ponts et Chaussées, 1848, II	The surface, mid-depth and bottom quantities are in the ratio of the numbers 100, 141 and 125.		
Elbe at Harburg—Blohm	1837–54	Zeitschrift des Arch. und Ing. Ver., Hanover, 1867	Surface mid-depth and bottom quantities are in the ratio of the numbers 100, 93.8 and 98.2.		

A study of the table shows only one case in which the sediment per cubic unit is not greater at the bottom than at the surface. This is that of the Elbe at Harburg. There are two cases where the mid-depth amounts are less than the surface values, the Mississippi at Winona and the Elbe at Harburg. There are three cases where the bottom values are less than those at mid-depth, the Sacramento at Kerscheval's, the Mississippi at Clayton, and the Garonne.

In all other cases there is a marked increase from surface to bottom. When it is considered how extensive a range of measurements is represented in these means and due weight is given to the careful and far-

reaching observations of the Mississippi and Missouri Commissions, it is thought that the law of increase from surface to bottom may be considered as established.

The table shows two other facts. *First*, that surface observations are not an accurate index of the mean amount of sediment. *Second*, that no general coefficient should be used to reduce surface observations to mean values for all depths, since this coefficient will vary in different rivers and for different stages of the same river.

It was hoped that data might be found from which a relation could be established between the form of the velocity and sediment curve in the same vertical. Measurements at all points of the vertical are needed with simultaneous velocity observations at the same depths. The only ones obtained at all partaking of this nature are those in the Sacramento River, which are too limited to be of service.[1] The observations at Carrollton in 1880 showed little change in the surface sediment with change of stage.

Law of Distribution in the Horizontal.—In this study the data available are still more limited. Of the extensive measurements carried on by the Mississippi and Missouri Commissions, the author can find no single case where the samples from the eight positions in the transverse sense were kept separate and the weights published. Mr. Seddon[2] states that nothing of special interest was shown by these measurements at St. Charles, so the data were not published. The Carrollton observations[3] of 1880 are said to have shown a uniform distribution from side to side of the channel and so are not published. Major Cunningham[4] obtained no data from which a law could be predicated in his measurements on the Ganges Canal.

Those at Columbus in 1858 are not printed in detail, but the Carrollton observations of 1851–52 give the single satisfactory set. The measurements made in 1879 by Mr. McMath at St. Louis are not published in detail, but give some facts of interest. The last two sets are represented in the table on the next page by their means for such positions as are indicated.

[1] An article by C. C. Babb, Jun. Am. Soc. C. E., abstracted in *Engineering News*, August 10th, 1893, is said to give curves representing sediment at different depths in the Potomac River.

[2] "Report of the Chief of Engineers, United States Army," 1887, IV, p. 3090.

[3] "Report of the Chief of Engineers, United States Army," 1883, III, p. 2216.

[4] *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 34, "Roorkee Hydraulic Experiments."

DISTRIBUTION OF SEDIMENT IN THE HORIZONTAL.

Place of observation and observer.	Date.	Reference.	Parts of Sediment in 1 000 000 Parts of Water at the Positions Indicated.		
			Bank.	Middle.	Bank.
Mississippi at Carrollton —Forshey	1851–52	Rep't on Miss., Humphreys and Abbot, 1861	542 (300 ft. from e. bank).....	573	543 (400 ft. from w. bank).
Mississippi at St. Louis — McMath	1879	*Van Nostrand Eng. Mag.*, 1883.	Mean of sediment in the river, 2 062 parts; mean of sediment, 281 ft. from Mo. side, 2 736 parts; mean of sediment 1 291 ft. from Ill. side, 1 523 parts.		

These meager measurements are offered rather to show the need of attention to this matter than as a proof that the maximum of sediment is near the thread of the stream. Mr. McMath[1] found the maximum in nearly every case several hundred feet on the Missouri side of the line of maximum velocity. This could, perhaps, be accounted for by the fact of the Missouri water being more highly charged since its sediment is finer. If the Ohio and Mississippi flow side by side without mingling their waters[2] it may be that a similar phenomenon occurs at St. Louis.

Surface Convexity and the Lateral Movement of Suspended Matter.— It has been stated[3] that crevasses in the Mississippi show a marked swelling or convexity at the thread of the current, where it crosses the levee, and that this convexity, which is due to the excess of velocity, has the effect of drawing floats to a narrow line at the filament of maximum velocity.

Major Cunningham[4] could measure no sensible curvature at the center of the Ganges Canal. He had expected to find such a swelling on the ground that increased velocity would be accompanied by decreased pressure. He quotes the statement of General Rundall that the surface of the Godavery and Mahanuddy Rivers was obviously con-

[1] "Silt Movement by the Mississippi," R. E. McMath. *Van Nostrand's Engineering Magazine*, Vol. 28, 1883, p. 36.

[2] See "Report on the Mississippi," Humphreys and Abbot, 1876, p. 136. Compare, also, the statement made by McMath in "The Mississippi as a Silt-Bearer," *Van Nostrand's Engineering Magazine*, 1879, p. 222. The Missouri water was found to contain 2½ times the amount of sediment carried by the water of the upper Mississippi where the two streams were running side by side, with a common velocity, past Bissell's Point.

[3] "Report on the Mississippi," Humphreys and Abbot, 1876, p. 284.

[4] *Proceedings* of the Institution of Civil Engineers, Vol. 71, pp. 11 and 12.

cave, plane or convex, according as the rivers were falling, stationary or rising.

Mr. Flamant[1] shows that there should be no such difference in pressure since a fluid transmits pressure equally in all directions and the surface should be level for permanent motion. He explains the convexity measured by Baumgarten[2] in the Garonne in one case on the ground that it did not correspond to a permanent state, but to a condition of rise when the center would show the increase before the sides.

M. Baumgarten[3] concludes from his two measurements on the Garonne that the changes are scarcely sensible.

The experiments of Darcy and Bazin[4] showed no results which could be said to express a law.

Prof. De Volson Wood[5] states that the water is highest where the velocity is greatest, but gives no substantiation for the statement.

M. Debauve[6] accepts the idea as true and explains its apparent disagreement with the hydrostatic law on the ground of viscosity, the water tending to the form which will offer the least resistance at the banks.

Major Cunningham[7] found that surface floats placed near the banks were uniformly drawn out to the thread of the stream, while subsurface floats maintained a direction sensibly parallel to the banks.

Mr. McMath[8] holds that suspended material is borne from the sides toward the center. The same statement is made by Dupuit[9] with reference to bodies floating at the surface.

M. Lagrene[10] considers the upper surface in straight sections to be sensibly horizontal so far as present knowledge reaches.

In general it may be said that the movement of surface floats shows a tendency toward the line of maximum velocity. That the difference

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1 See *Annales des Ponts et Chaussées*, 1882, IV, p. 56. Also *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 66.

2 See *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 12, or *Annales des Ponts et Chaussées*, 1848, 2, pp. 28–30.

3 *Annales des Ponts et Chaussées*, 1848, 2, p. 30.

4 See "Recherches Experimentales." Darcy and Bazin. Plates XIX to XXVI. Also *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 12.

5 *Van Nostrand's Engineering Magazine*, 1879, p. 370. He claims that the cause lies in the reduction of pressure with increase of velocity.

6 "Navigation Fluviale et Maritime." V. Debauve.

7 *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 23. "Roorkee Hydraulic Experiments."

8 *Van Nostrand's Engineering Magazine*, Vol. 28, p. 35.

9 "Etudes sur le Mouvement des Eaux," p. 218.

10 "Cours de Navigation Intérieure," p. 53.

in height of surface between center and sides is a concomitant phenomenon cannot be said to be proved. There are many statements that the fact exists, but such measurements as are available fail to substantiate them.

Does Suspended Matter Move Faster Than the Current?—The question of the existence of a relative velocity between surface floats and the surface current may be said to be settled. It has been considered a source of error in the measurement of surface velocities by floats for some time past. The velocity of the float is greater than the mean velocity of the displaced water. For the same float, this relative velocity will increase with increase in depth of flotation.[1]

Major Cunningham[2] takes the contrary view, claiming that the velocity of a submerged rod is somewhat less than the mean velocity past its immersed length, so that it should only extend $\frac{94}{100}$ of the total depth to the bottom in order to give a true indication of the mean velocity in a vertical.

The experiments of M. Du Boys[3] upon the Rhone may be said to settle the matter beyond dispute. He found from experiments upon boats that they moved sensibly faster than the current, and that this relative velocity varied with the form of the boat and increased with its size and with the velocity of the current.

Experiments by M. Bérard,[4] in 1886, on an artificial canal, showed that a float moved faster than the surface velocity, but almost identically at the same rate as the mean velocity of the displaced water.

M. Du Boys,[5] however, gives the case of a boat which had a velocity of 4.46 m. in a current whose surface velocity was only 2.75 m. These differences are too great to be attributed to the fact that the maximum velocity is below the surface.

A block of wood and a floating canal-boat will not remain side by side in a river, but gradually draw away from each other, the boat taking the lead.

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[1] Noted and incorrectly explained in *Zeitschrift des Architekten- und Ingenieur- Vereins*, Hanover, 1887, p. 628.

[2] "Roorkee Hydraulic Experiments." *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 22.

[3] "La Marche des Bateaux dans les Courants Rapides." *Annales des Ponts et Chaussées*, 1886, I, pp. 199–242.

[4] "Marche des Flotteurs dans les Courants," *Annales des Ponts et Chaussées*, 1886, II, p. 30.

[5] "Hydraulique." Flamant, p. 299.

The Question of Lateral and Vertical Flow in Rivers.—Islands are said to be formed at the center of rivers at the expense of the banks, indicating a transverse flow of the particles toward the middle. Is there a regular lateral flow of the water obeying a law as fixed as that which determines the general law of translation ? If so, it will enter as an important factor in governing sedimentary movements. Prof. James Thomson[1] has demonstrated the presence of such a flow at curves where the motion at the surface is outward and at the bed is inward. This offers a suitable explanation of the cause of shoals opposite sharp bends. In a regular channel centrifugal force does not enter in to produce these effects. Are they present ? Mr. McMath says[2] that observation has failed to detect any division of a stream into ascending or descending areas other than local motions due to eddies. Major Cunningham[3] claims to have detected a surface flow toward the center in the Ganges Canal as indicated by the motion of the floats near the banks, while a sub-surface flow in the contrary sense was indirectly shown by the fact that the deeply immersed floats showed no general transverse tendency.

Experiments made by J. B. Francis[4] in regular canals with whitewash discharged through a tube opening near the bed, seemed to show a vertical movement of the water from the bed to the surface, the whitewash appearing at a distance down-stream varying from 10 to 30 times the depth. Mr. Francis offers this vertical movement as the cause of suspension of sediment.

Prof. De Volson Wood,[5] upon the ground of these observations of Mr. Francis, adopts the idea of a lateral surface movement toward the banks, with a corresponding bottom current toward the center.

F. P. Stearns, M. Am. Soc. C. E.,[6] on the other hand, holds the view that there is a motion of bottom water to the surface at the sides and toward the center at the surface.

The observations of Mr. Francis might seem to be corroborated by the positive statement[7] that the Mississippi water is constantly rising from the bed to the surface.

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[1] "Encyclopedia Britannica." Article Hydrodynamics, p. 498.

[2] "Silt Movement by the Mississippi," *Van Nostrand's Engineering Magazine*, Vol. XXVIII, p. 35.

[3] *Proceedings* of the Institution of Civil Engineers, Vol. LXXXII, pp. 23, 24. "Roorkee Hydraulic Experiments."

[4] *Transactions* of the American Society of Civil Engineers, Vol. VII, p. 109.

[5] *Van Nostrand's Engineering Magazine*, Vol. XXI, 1879, p. 369.
Also *Transactions* of the American Society of Civil Engineers, June 17th, 1879.

[6] *Transactions* of the American Society of Civil Engineers, Vol. XII, p. 331.

[7] "Report of the Chief of Engineers, United States Army," 1883, III, p. 2218.

5. Solid Discharge of Rivers.

In many articles on river correction and in most of the treatises on river hydraulics will be found statements of the amount of silt carried by streams in various parts of the earth. The amounts are unreliable in some cases and in many are expressive of only suspended matter to the exclusion of the more or less extensive movement along the bottom. A collection of such non-homogeneous data is not deemed of value for the purposes of the present paper.[1]

Proportionate Amounts Suspended and Dragged.—The quantitative relation existing between the amounts moved at or near the bottom and those carried in free suspension has offered another opportunity for diversity of opinion.

Dupuit[2] looks upon transportation in suspension as the most important element, on the ground of the slight velocity of the materials dragged. Prof. Forshey[3] concludes that the matter pushed along the bed of the Mississippi forms about three-fourths of its total solid discharge.

M. Guérard[4] is persuaded that the greater portion of the solid matter discharged by the Rhone is pushed along its bed. The Mississippi River Commission,[5] after a series of careful measurements of sand waves at Carrollton, decided that not more than 0.8 of 1 per cent. of the total solid discharge was moved along the bed of the river at this point. Major Ruffner[6] states that the movement of material at the lower laminæ of the Missouri at St. Charles was believed to be as great as that in all the rest of the river. He refers[7] also to observa-

[1] The following list gives a few sources of information upon this point:

"Report on the Mississippi," Humphreys and Abbot, 1876, p. 146.

Zeitschrift des Arch.- und Ing - Vereins, Hanover, 1867, pp. 245–50.

Proceedings of the Institution of Civil Engineers, Vol. XXI, pp. 15, 27, 459; Vol. LIII, p. 18; Vol. LI, pp. 216, 217; Vol. LVII, pp. 272–4.

The Engineer, October 25th, 1889, p. 343.

Annales des Ponts et Chaussées, 1848, II, pp. 46–48; 1860, I, p. 137; 1860, II, p. 374; 1869, I, p. 588; 1871, I, p. 15.

"Canal and River Engineering," Stevenson, p. 318.

"Improvement of Non-Tidal Rivers," Ruffner.

"Cours d'Hydraulique Agricole et Urbaine." M. Bechmann, 1895, pp. 109, 120. Also treatises on general geology.

[2] "Etudes sur les Mouvements des Eaux," p. 216.

[3] *Proceedings* of the American Association for the Advancement of Science, Nashville, 1877. See also *Van Nostrand's Engineering Magazine*, Vol. XX, p. 227.

[4] *Proceedings* of the Institution of Civil Engineers, Vol. 82, p. 309.

[5] "Report of the Chief of Engineers, United States Army," 1883, III, p. 2218.

[6] "Improvement of Non-Tidal Rivers," p. 78.

[7] The same, p. 138.

tions at Lake Providence, La., which indicated that a large amount was moving along the bed of the Mississippi at that point of which the moving sand waves represented only a small portion.

Captain Eads and Mr. Corthell[1] join with the Mississippi River Commission in the statement that nearly all the solid matter in the Mississippi is carried in suspension, while but a small proportion is dragged on the bottom.

It is believed that this last statement represents the actual case in most sedimentary rivers.

6. Tabulation of Observed Data on Dragging.

A tabulation of the best known results of experiment on velocities at at which dragging begins is given on the next page. Those given by Bouniceau appear to be taken from the results of Dubuat's and Telford's experiments. The others are believed to represent original measurements or computations. In some cases the published results do not state whether the velocity measured was that at surface, mid-depth or bottom.

Further statements of velocities are given by Weisbach, Unwin, Church, Bechmann and others, but they are all based upon the measurements detailed in this table. A limited collection of similar data is given in the Report of the Chief of Engineers, U. S. Army, 1885, I, pages 569–570.

The detail of Dubuat's and Sainjon's measurements has been given in a preceding part of this paper. The measurements in the Upper Rhine were made near Alt-Breisach, with a smooth river-bed. In the cases so marked, motion did not take place until the particles were subjected to a slight disturbance from the outside.

Login's experiments were made with a stream averaging ½ in. in depth. Telford's velocities are those at which erosion begins. Blackwell's measurements have been referred to on page 248.

The materials are given as described by the experimenter. A careful set of measurements of these velocities, made with improved apparatus, is much needed.

Displacement of Crests of Sand Waves.—For purposes of comparison, a collection has been made on page 310 of measurements on sand waves.

[1] *Transactions* of the American Society of Civil Engineers, Vol. XI, p. 262.

TABLE GIVING VELOCITIES OF CURRENT AT WHICH DRAGGING BEGINS.

Authority	Dubuat.	Telford.	Blackwell.	Sainjon.	Login.	Rhine measurements.	Zschokke.	Verein Hütte.	Bouniceau.
Reference	Hydraulique. Paris, 1786. p. 94.	Partiot in Annales des Ponts et Ch. 1871, I, p.34.	Proc. Inst. Civil Engineers, Vol.82, pp.47–50.	Partiot in Annales des Ponts et Ch. 1871, I, p.33.	Stevenson's Canal and River Engineering, p. 315.	Zeitschrift des A. und I. Vereins zu Hannover, 1884, p. 176.	Lectures, Zürich, 1895.	Ingenieurs Taschenbuch.	"Etudes sur la Navigation," 1845, p. 19.
REMARKS.	Bottom velocity. Feet per sec.	Bottom (?) velocity. Feet per sec.	Feet per second.	Bottom velocity. Feet per sec.	Bottom velocity. Feet per sec.	Feet per second.	Feet persecond	Bottom velocity. Feet per sec.	Bottom velocity. Ft. per sec.
Soft earth		0.25							
Brick clay (allowed to settle ½ hour in water)					0.25				0.26
Potter's clay	0.27							0.26	
Soft clay		0.50						0.52	0.49
Fresh-water sand					0.667				
Large sand	0.71								0.72
Vegetable soil					0.833				
Sand		1.00							0.98
Firm sand								1.02	
Sea sand					1.103				
Gravel (size of anise seed)	0.36								0.36
Gravel (size of peas)	0.62				2.000	2.46 (if disturbed)	0.62		
Gravel (size of beans)						2.95 (if disturbed)	1.066		
Gravel (diameter, .008 ft.)				0.82					
Gravel (diameter, .03 ft.)				1.64					
Brickbat (contents, 2.59 cu.ins.)			1.75–2.00						
Gravel		2.00							
Oolites (2.39 cu. ins.)			2.00–2.25						
Slate (2.38 cu. ins.)			2.00–2.25						
Sea pebbles (1.06 ins. diameter)	2.13							2.30	2.13
Brickbats (4.76 cu. ins)			2.25–2.50						
Flints (1.96 cu. ins.)			2.50–2.75						
Brickbats (18.5 cu. ins)			2.75–3.00						
Oolites (17.68 cu. ins)			2.75–3.00						
Slate (9.06 cu. ins.)			2.75–3.00						

Flints (10.37 cu. ins.)			3.00–3.25						
Pebbles		3.00							3.28
Boulders								3.08	
Gravel (.085 ft. diameter)							3.09		
Gravel (.13 ft. diameter)				3.28					
Gravel (size, hazel to walnut)						3.48 (if disturbed)			
Gravel (size, pigeon's egg)						3.67 (if disturbed)			
Smallest gravel						3.87			
Gravel (walnut size)						4.92			
Gravel (.17 ft. diameter)							5.15		
Gravel (weight, 2[illegible]0 grams)						4.92 (if disturbed)			
Broken stone		4.00							3.94
Gravel (weight, 2 500 grams)						5.90 (if disturbed)			
Conglomerate (soft schist)		5.00						4.90	5.00
Gravel (.33 ft. diameter)				4.92					
Laminated rock		6.00						6.00	6.00
Gravel (.56 ft. diameter)				6.56					
All gravel						6.56 (if disturbed)			
Gravel (.18 cu. ft.)							7.22		
Gravel (1.25 ft. diameter)				9.84					
Hard rock		10.00						10.36	9.84
Gravel (1 09 cu. ft.)							10.29		
Gravel (2.20 ft. diameter)				13.12					
Gravel (2.18 cu. ft.)							15.44		

Tabulated Data on Movement of Sand Waves.

Locality.	Date.	Observer.	Velocity of Stream. Feet per Second. Surface.	Mean.	Bottom.	Average depth of stream. Feet.	Material.	Dimensions of Wave. Feet. Height.	Length.	Velocity of crest. Feet per day.	Reference.
Mississippi, at Helena	March to June, 1879.	J. B. Johnson		3.58		13 to 30	Sand.	5.0	328.0	17.75	Rep't Chf. of Engrs., U. S. A., 1879, p. 1967.
Loire, at Angers	June to Nov., 1858.					Scale read from − 0.3 to + 0.85.	Sand.			7.35	Debauve Nav. Fluviale, p. 135.
Loire, at Angers	Nov., 1858, to June, 1859.					Scale read from + 0.80 to + 3.75.	Sand.			29.52	
Mississippi, at Lake Providence	Nov., 1879, to Nov., 1880.	Hider				(High water) 76.	Sand.	13.0	470	22.5	Rept. of Chf. of Engrs., U. S. A., 1883, pp. 2197–98, Appendix S. S.
"	"	"				(25 ft. below high water) 46.	"	6.6	252	9.7	
"	"	"				(High water) 80 to 100.	"	6.9	332	42.	
"	"	"				(25 ft. below high water).	"	7.6	325	10.8	
Sacramento, at Kerscheval's	March, 1879	Le Conte	5.50		4.48 (2 to 7 ft. above bottom).	20 to 25 ft.	Sand.	2	13		Report of Chf. of Engrs., 1879, p. 1759.
Garonne, at Col-de-Fer	1840–43	Baumgarten		6.5 to 8.0			Gravel.	4.2		82 (ft. per year).	Annales des Ponts et Chaussées, 1848, 2.
Loire	1858–59	Partiot					Sand.			36.0	Sables de la Loire. Partiot. Annales I, 1871.
"	"	"					"			16.4	
"	"	"					"			16.4	
"	"	"					"			6 56	
Loire, at Nantes	1858	Duval					Sand.			4 2	Partiot. Annales des Ponts et Chaussées, 1871, I, p. 43.
"	"	"					"			11.2	

Additional measurements, made by M. Sainjon, are given at page 257 of this paper.

In general, it may be said that the displacement of these wave crests increases with the stage and with the velocity, at least up to a certain point of bottom velocity. Their form and motion is most regular at times when the river is neither rising nor falling. Their motion is most rapid when the river is rising. They are largest in deep water.

7. Velocities of Vertical Current Required to Keep Particles Suspended.

A particle whose specific gravity is greater than 1 will sink in quiescent water when viscosity is overcome. It will, when falling freely from rest, after a very short period of acceleration, attain a state of velocity practically uniform. The same particle should be kept suspended indefinitely by a constant current, whose vertical component has a velocity equal to that asymptotically approached by the falling particle.

Experiments have been made to determine the velocities of currents necessary to keep particles of various sizes and specific gravities suspended when acting in a direction opposed to gravity.

The table[1] on page 312 gives the results of experiments made by M. Thoulet. It represents the maximum velocities in millimeters per second of, and the thrusts in milligrams exercised by, currents capable of holding suspended, at a fixed point in a tube, spherical grains of radius varying from 0.1 to 2.5 mm., and of densities, in air, between 1.5 and 4.0. The velocities are taken from diagrams constructed from results of actual experiment. The thrusts P are computed on the supposition that a current with a velocity v, in millimeters, which holds in suspension a body of density γ', exercises against the body a thrust

$$P = \frac{4}{3}\pi r^3 (\gamma' - 1)$$ (in milligrams, where r is expressed in millimeters).

The range of values for (r) and (γ') is sufficient to cover all ordinary mineral grains.

There can be little doubt that the velocity of vertical current or the amount of vertical thrust required for the suspension of ordinary grains is very slight. Col. Mansfield reports[2] observations on grains of sand allowed to fall from rest in water. The maximum velocity

[1] For further details see p. 269; also *Annales des Mines*, 1884, I, p. 521.

[2] "Report of Chief of Engineers, United States Army," 1886, pp. 1298–1299.

Radius (r) of grains in millimeters.	Velocity (v) of the Current in Millimeters per Second. Thrust (P) in Milligrams Exercised Against the Spherical Grain of Density (γ').											
	$\gamma' = 1.5$ v	P	$\gamma' = 2.0$ v	P	$\gamma' = 2.5$ v	P	$\gamma' = 3.0$ v	P	$\gamma' = 3.5$ v	P	$\gamma' = 4.0$ v	P
0.1	13	0.0002	23	0.004	32	0.006	40	0.008	47	0.01	53	0.01
0.2	23	0.002	40	0.03	55	0.05	69	0.07	79	0.08	85	0.10
0.3	32	0.006	57	0.11	78	0.17	94	0.23	106	0.28	116	0.34
0.4	40	0.01	70	0.27	94	0.40	114	0.54	129	0.67	143	0.80
0.5	51	0.03	82	0.52	107	0.78	132	1.05	151	1.31	168	1.56
0.6	57	0.04	92	0.90	120	1.36	147	1.81	168	2.26	187	2.72
0.7	61	0.07	100	1.44	133	2.15	163	2.87	185	3.59	205	4.30
0.8	65	0.11	108	2.14	142	3.22	174	4.29	198	5.36	221	6.44
0.9	68	0.15	116	3.05	152	4.58	185	6.11	211	7.63	237	9.16
1.0	72	0.21	123	4.19	161	6.28	195	8.38	224	10.47	252	12.56
1.1	75	0.28	128	5.57	169	8.36	203	11.15	234	13.94	264	16.72
1.2	78	0.36	133	7.24	175	10.86	211	14.48	243	18.10	273	21.72
1.3	80	0.46	137	9.20	180	13.80	218	18.40	252	23.01	281	27.60
1.4	82	0.57	141	11.49	184	17.24	224	22.99	260	28.73	288	34.48
1.5	84	0.71	144	14.13	188	21.21	229	28.27	267	35.34	295	42.42
1.6	86	0.86	147	17.15	192	25.74	234	34.31	271	42.89	300	51.48
1.7	88	1.03	149	20.57	195	30.87	237	41.16	274	51.45	304	61.74
1.8	90	1.22	151	24.42	197	36.64	240	48.86	277	61.07	308	73.28
1.9	92	1.44	153	28.72	199	43.10	243	57.46	280	71.83	312	86.20
2.0	94	1.67	154	33.50	201	50.27	246	67.02	283	83.78	315	100.54
2.1	95	1.94	155	38.78	202	58.19	249	77.58	286	96.98	317	116.38
2.2	96	2.23	156	44.59	203	66.90	251	89.20	288	111.50	319	133.80
2.3	97	2.55	156	50.95	204	76.45	253	101.93	290	127.41	321	152.90
2.4	98	2.89	156	57.89	205	86.86	254	115.81	291	144.76	323	173.72
2.5	98	3.27	156	65.44	206	98.17	255	130.90	292	163.62	324	196.35

reached at the bottom showed that a vertical current of 1 or 2 ins. per second would keep them suspended.

Mr. T. E. Login,[1] found that the following materials had a rate of sinking in water of, and consequently would be sustained by a vertical current of, a velocity equal to the numbers given below:

Materials.	Velocity required. (Feet per second.)
Brick clay (mixed with water and allowed half hour to settle)	0.009
Fresh water sand	0.166
Sea sand	0.196
Rounded pebbles (size of peas)	1.000

Diagrams have been prepared by Richards[2] and Woodward to show the relations existing between the specific gravity, diameter and velocity of fall in water of various minerals. They are constructed from a formula given by Rittinger.[3]

$$V = 2.44 \sqrt{D(\delta - 1)}$$

where V = velocity of fall in meters per second.

D = diameter of particles in meters.

δ = specific gravity of the mineral.

These diagrams are extended to cover specific gravities between 1 and 15, diameters between 0 and 0.06 m., and velocities of fall up to 2.3 m. per second.

8. Results Expressed in the Form of a Series of Propositions.

The following propositions are believed to express the main facts, so far as they are known at present, which must be recognized by any broad theory of the cause of the suspension of sediment:

1. The movement of solids by water currents may take place by dragging, by intermittent suspension or by continuous suspension.

2. Motion in each of the three ways is increased with increase of depth; yet the depth itself can only affect the intermittent motion.

3. Motion by each of the three ways is increased by increase in mean velocity.

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[1] *Proceedings* of the Royal Society of Edinburgh, Vol. III, p. 475.
Also "Canal and River Engineering," Stevenson, p. 315.

[2] "The Velocity of Bodies of Different Specific Gravity Falling in Water." Richards and Woodward. *Transactions* of the American Institute of Mining Engineers, 1890, Vol. 18, pp. 644–648.

[3] Rittinger's "Aufbereitungskunde," 1867, p. 195..

4. The presence of sediment in a stream decreases its mean velocity.

5. Dragging as well as suspending power increases with the heaviness of the liquid and with its coefficient of viscosity.

6. By far the greater amount of solid material transported by sedimentary rivers is carried in continuous suspension.

7. Sand waves of considerable size are formed in the larger rivers. Their motion down stream is slow, increasing in rate up to a certain critical value of bottom velocity and then decreasing with further increase.

8. The fineness of material in suspension increases from the rise to the embouchure of rivers.

9. The weight of suspended material per cubic unit of water decreases from rise to embouchure.

10. Increase in vortex motion increases power of transport.

11. Continuous vertical or lateral currents which could not be explained by local causes in rivers have not yet been proved.

12. The phenomenon of suspension requires for its explanation a continuous upward force. Intermittent forces are not sufficient, although the intensity of the force may vary.

13. Bodies suspended in flowing water, either intermittently or continuously, tend to acquire a velocity greater than that of the water surrounding them.

14. The transportation of material consumes part of the energy of any silt-bearing stream. A greater load of fine particles than of coarse can be carried with the same expenditure of energy.

Part III.—Discussion of Theories of Suspension.

The explanations offered as to the cause of this phenomenon, as detailed in the preceding pages, fall naturally into four categories. It will be well to examine each in the light of the facts at hand before drawing conclusions as to their relative value.

(a) *Theory of a Continuous Upward Flow.*—J. B. Francis[1] Past-President Am. Soc. C. E., sought to explain the suspension of sediment in streams by the presence of a continuous upward flow of the water at the bed. This flow was believed to be proved by experiments made by liberating whitewash at the bed of two different channels and noting its appearance at the surface at distances down-

[1] *Transactions* of the American Society of Civil Engineers, May and June, 1878.

stream varying from 10 to 30 times the depth. The experiments of M. Thoulet[1] have shown how slight an upward current is required to produce this effect. Were such a resultant upward current proved, it might offer a satisfactory explanation of the phenomenon. There are, however, certain objections to this view which seem to render it inadmissible. Considering a long stretch of river, a steady flow from bed to surface at the center must be accompanied by a corresponding flow outward to the sides and a downward reverse flow along the bed from the sides to the center. Such a flow has not been observed. On the contrary several writers[2] have advanced the idea of a flow in the contrary sense, though this, too, must be regarded as yet unproved.

Again, even were the flow from bed to surface at center and outward to the sides proved, it would constitute no explanation of continuous suspension, unless the velocities of flow at center and sides were unequal, since it would give no resultant upward thrust. The algebraic sum of the vertical components of all the internal movements of the liquid should reduce to zero in the stretch considered.

Proofs of flow from bed to surface or from sides to center based upon indications offered by floating substances—the case in both Mr. Francis' and Major Cunningham's experiments—can not be considered conclusive. In the one case the movement might be attributed to local currents induced by the presence of the experimental apparatus. In both cases, the fact that a floating body tends to move faster than the surrounding medium and would follow the line of least resistance would cause floating substances to indicate currents which did not actually exist. This matter will be referred to later.

(*b*) *Discontinuous Upward Flow or Eddy Theory.*—There are certain writers who attribute the entire suspending power of streams to vertical currents incident to the complex eddying motion induced by the asperities of the bed. The numerous valuable articles[3] by R. E. McMath, M. Am. Soc. C. E., have taken the strongest position on

[1] See p. 270.

[2] See *Proceedings* of the Institution of Civil Engineers, Vol. 71, p. 23 and discussion. Also *Transactions* American Society of Civil Engineers, Vol. 12, p. 333.

[3] "Silt Movement by the Mississippi." *Van Nostrand's Engineering Magazine*, Vol. 28, 1883, p. 32.

"The Mississippi as a Silt Bearer." *Van Nostrand's Engineering Magazine*, Vol. 20, 1879, p. 218.

"Theory and application of the Permeable System of Works for the Improvement of Silt-Bearing Rivers." *Engineering News*, November 1st, 1879.

this line. Upon the basis that the fact of suspension itself is an evidence of a resultant upward thrust, Mr. McMath states his position in these words:[1]

"This third hypothesis goes but one step farther in ascribing to the irregular movements the whole work of suspension upon the ground that a cause known to exist wherever the fact to be accounted for occurs, and admitted to be efficient, must be considered the sole cause unless a co-working agency is known, or the cause is insufficient to produce the observed result. Observation readily detects whirls, boils and eddies in the act of bringing water and suspended material from the bottom to the surface and laterally from the sides to the center of the river. Observation has never detected any other cause incident to the flow of streams which produced these effects."[2]

The efficiency of vortices in producing local results has passed beyond the range of controversy. M. Boussinesq[3] divides them into two classes; first, those in which the constituent water is constantly changing as the result of an axial flow which increases in velocity as the radius of curvature decreases; second, those in which the constituent water remains the same, being formed during an interval of time by a definite mass of fluid and manifesting themselves in a complex rotation.

A case of the first kind is the common vortex over the discharge pipe of a hand basin. One of the second is seen in the experiment of the revolving glass or basin of water described earlier in this paper in the experiments of M. Fargue and M. Gallois.

In both forms of vortex the tangential velocity increases with approach to the axis in contradistinction to the case of a rotating solid. In each case solid material obeys an impulse toward the axis whether there be a flow of liquid in that direction or not. This is due to the tendency of the solid to move faster than the liquid and in so doing to follow the line of least resistance, *i. e.*, the line where differences of velocity are least. In the first form an additional impulse toward the axis is received from the current setting in that direction. These phenomena are clearly shown in the experiments referred to, though

[1] "Silt Movement by the Mississippi." *Van Nostrand's Engineering Magazine*, Vol. 28, p. 35.

[2] "The suspension of matter involves an upward motion of the water or medium in which such matter is suspended, if in no other way, by the law of adhesion. * * * Upward motion of the suspending medium involves of necessity a downward movement of equal volume, but not necessarily of equal velocity." McMath in *Engineering News*, November 1st, 1879.

[3] "Essai sur la Théorie des Eaux Courantes," p. 616.

M. Gallois'[1] explanation is at variance with that given above. He takes the position that the particles are impelled toward the center because the relative velocities are greatest at that point. This is incompatible with the theory that the standard form of a vertical velocity curve in streams is a parabola. If the friction initiated at the sides exercises its retarding effect progressively toward the center, as is believed to be the case in rivers, the absolute velocity near the axis of the vortex should be greater, but the relative velocities of the fluid filaments should be a maximum near the sides. In other words the solids would move toward the axis of the vortex with radial velocities varying directly with the distance from the axis. According to M. Gallois' explanation this relation would be an inverse one.

In a flowing stream these eddying movements are present in the forms described and in numerous intermediate states. Irregularities in the bottom sometimes send currents to the surface in the form of boils where there appears to be only a swift vertical velocity without much rotation. In the complex motion of the stream these visible disturbances are but the type of an infinite variety of similar movements taking place between small groups of molecules. These are not so evident to the eye, but consume in the aggregate a large proportion of the stream's energy. The asperities of the bed and banks send off these rotary groups with axes inclined at all angles and with velocities of translation having a resultant in the stream direction. Whatever resistance is offered along the air profile will manifest itself in complicating the direction of these movements of translation. The vital question is to know the direction of the resultant motion. Has it a component acting upward ?

It is seen at once that vertical motion of the water itself can only be a local phenomenon. The resultant of all the external forces acting upon a given stretch of river will have a direction down stream, and will approach zero as the motion approaches uniformity. Its vertical component will be downward, not upward. The same thing will be true of all the complex motions in the interior of the liquid prism considered in the aggregate. It is true, however, that the reverse downward current need not equal in velocity the original upward one.[2] The diminution in velocity may be counterbalanced

[1] See *Engineering News*, March 23d, 1893.

[2] This fact is noted, without comment, by Mr. McMath in *Engineering News*, November 1st, 1879. It is thought that he was the first to suggest it in connection with suspension.

by an increase in the moving area. This is an important point affecting the power of suspension. While there is no resultant vertical motion of the water, there is a resultant vertical thrust exercised upon sediment in the aggregate and acting as an efficient cause of suspension.

The thrust exercised by a current upon an immersed solid is

$$P = k\ F\ \gamma\ \frac{v^2}{2g}$$

and obviously varies with the square of the velocity. An eddy, having an upward component of velocity represented by 4, would exert an upward thrust, available for suspension, proportional to 16. In descending, the same water might cover twice the area, and have a downward velocity of 2. Then the corresponding downward thrust would be proportional to 8. The resultant of such a local system would be an upward thrust available for continuous suspension. In addition to this, particles carried by the vortex to the surface, and especially when freed at that point from its influence, would exhibit the phenomenon of temporary suspension, before gravity had again brought them back to the river bed. Cover the stream with systems of this character, and there results a modified form of what is actually seen. Eddies are induced by the rugosities met with in the earth and air profiles. The abrupt changes in the bottom, and its relatively unyielding character, cause the upward currents to have the greater velocity in general, though they must of necessity be restricted in relative extent.

The fact that the descending areas must be much greater than the ascending ones in order to give rise to this force shows that, in the major part of the stream, an active force is at work tending to carry material to the bottom, aside from the force of gravity acting upon the particles. This suggests, though it does not prove, the existence of another cause of suspension, and such a cause is actually found, as shown later.

Here, then, in obedience to the law which governs the thrust exercised by water impinging upon a solid, there is a resultant force acting upward in flowing streams, and capable of performing an efficient part in the continuous suspension of solid matter. While the aggregate of all the interior motions of the fluid can have no upward resultant, still the thrusts exercised upon solid suspended matter by these motions can and do have such a resultant as shown.

It is evident that this force would disappear in circular conduits, flowing full, since a uniform profile may be assumed. The eddies would be thrown off with like intensity at all points toward the center.

M. Boussinesq[1] attributes increase in vortical motion at the walls to any one of these causes; increased mean local velocity, increased roughness of lining or increase in hydraulic mean radius. The effect of the first two causes is evident. The influence of the increase of area per unit of wetted perimeter is thought to be shown by the greater field of action allowed for oscillatory movements perpendicular to the walls. These vibrations help in detaching groups of molecules and cause such groups to suffer sudden changes in intensity of tangential friction at the walls which, in turn, favors their formation into vortices.

In a circular conduit running full this would indicate an increase in vortical movements at the walls with increase in diameter. In passing from the walls to the center of the current, M. Boussinesq thinks the vortical agitation should increase where the wetted perimeter is concave, as in the case of closed conduits of rectangular or rounded forms, running full, because the field of action for each vortex is constantly narrowing and interference increases. On the other hand, this agitation should be less and be practically uniform for a rectangular cross-section of indefinite length of base. The agitation should be of about the same intensity in a large circular or rectangular conduit running full as when running half full. The action of the free surface in reflecting these movements ought not to differ widely from the action of the upper half of the section upon the lower half when running full.

Considered as a whole, the vortex theory,[2] as advocated by Mr. McMath, is entitled to a position as one of the major causes of suspension. The statement that it is the sole cause, and that observation has never detected any other cause incident to the flow of streams which has produced these effects would seem too broad a claim.

(c.) *Theory Based upon Eddies and Relative Velocities.*—Partiot was probably the first to give prominence to the suspending power of

[1] "Theorie des Eaux Courantes," p. 47.

[2] A valuable paper by William Starling, M. Am. Soc. C. E., Chief Engineer of the Mississippi Levee Commission, is in press at the present time, which brings this theory prominently forward. The term "vortex theory" is here used in a restricted sense, as indicated by the context.

eddies. He bases his explanation of suspension upon the presence, in these vortical movements, of exaggerated relative velocities which, in conjunction with the motion of the body, cause a thrust toward the most rapid filaments. The particles are then carried along with the motion of translation of the eddy.

Where eddies are not the acting force, he follows Dupuit's explanation as given hereafter.

(*d.*) *Theory Based upon Relative Velocities.*—This theory was brought forward by Dupuit. It utilizes the fact that all partially or wholly submerged bodies tend to move faster than the mean velocity of the displaced water. This makes a choice of path for relative motion necessary. The path chosen will be that which offers the least resistance, *i. e.*, diagonally toward the most rapid filaments.

Take the case of a floating homogeneous sphere. Let it have a velocity of translation in the stream direction equal to that of the mean of the displaced filaments. Assume uniform stream motion. Place it near the left-hand shore. The filaments of water at the right of the body are moving faster than its center of gravity, and by their friction tend to revolve it in an anti-clockwise direction. On the opposite side of the ball the friction is reversed in direction, and aids in producing the same rotation. What is the result upon transverse motion?

The ball will tend to place its center of gravity in the line of the resultant resistance of the filaments ahead of it. But in the theoretical case considered there is no resultant resistance upon the sphere, since it moves with the same mean velocity as the displaced water. The law stated above, then, does not call for any transverse motion under the conditions named.

Introduce the fact that the body has a velocity greater than that of the mean of the displaced filaments. A new element is introduced, for there is now a resultant resistance from the filaments ahead which is proportioned to the square of the relative velocity. This relative velocity, and consequently the resistance, increases until the resistance is equal to the accelerating force, when the motion becomes uniform. It is now incumbent upon the body to choose a path relatively to the fluid. It will tend to place its center of gravity in the line of action of the resultant resistance. This line of action is to the right of the center of gravity of the body. It will then take a diagonal path to the right, or toward the most rapid filaments.

The line of application of the mean resistance will coincide with the line of action of the mean velocity of the water immediately ahead. At any point of its path the mean resistance offered will be least, *i. e.*, the velocity of the body will be most nearly equal to the mean velocity ahead, if the center of the body is in the line of action of this mean velocity.

In summing up, then, Dupuit's theory of suspension rests upon the fact that submerged bodies tend to move faster than the mean velocity of the displaced water, and in so doing choose the line of least resistance.

It is the only satisfactory explanation yet offered of the phenomena shown in the experiments with rotating vessels, which are described by M. Fargue and M. Gallois. It offers an explanation of the increase in suspending power, with increase in depth and velocity. In the "Report on the Mississippi River" by Humphreys and Abbot (page 254) it is shown that the parameter of the horizontal parabola of velocities 5 ft. below the surface, at any stage, is proportional to the square root of the corresponding mean velocity. The same should be true of the vertical curve.

The reasonableness and value of this theory seem evident. Its incompleteness as an explanation of the whole phenomenon is shown by the fact that it takes no account of the suspending power of eddies and offers no explanation of the presence of sediment above the line of maximum velocity.

Before stating the conclusions to which this analysis of theories leads, it will be well to place emphasis upon a cause of suspension which has only been indicated in a general way.

Dupuit[1] suggested that the resultant of all the pressures upon a submerged body was greater in flowing or agitated than in quiescent water. He explains it on the ground of relative velocities of the filaments as detailed above.

M. Flamant[2] has adopted the same idea as an explanation of the tendency of surface floats to move faster than the current. His argument is that the floating body weighs more in agitated water than the weight of the water displaced, and that the excess shows its presence in the increased velocity. The explanation implied is that there is a resultant upward thrust in the surrounding fluid due to its agitation.

[1] "Etudes sur Le Mouvement des Eaux," 1848, p. 217.
[2] "Hydraulique," 1891, p. 298.

The explanation offered by Du Boys[1] for the increased velocity of surface floats has indicated the presence of a forward thrust, but has not called attention to the fact that this thrust has a direction above the horizontal. This follows from the discussion under (*b*) preceding. Experimental demonstration is wanting, but the fact that continuous suspension exists above the line of maximum velocity is an evidence that Nature shows the phenomenon indicated by theory.

It should be remembered that the line of maximum velocity has often been found at the surface. In such cases the suspending power due to the body's excess of velocity and tendency toward the line of least resistance acts from bed to surface in conjunction with the upward thrust due to the eddies. In the majority of cases it is found below the surface, and here a force must be acting downward to assist gravity in opposing suspension above the line of maximum velocity. This would indicate for such cases a decrease in suspended matter near the surface. The burden of the work then falls upon the upward thrust of the vortices. The fact that agitation increases from the line of maximum velocity to the surface tends to make the effect of this downward thrust disappear, since the latter decreases in intensity toward the vertex of the vertical curve. In this way marked differences in the intensity of sediment charge at the surface and at the line of maximum velocity are prevented. A difference in the amount carried at the surface ought to be shown according as the vertex of the curve of vertical velocities is raised or lowered—for instance, by the wind. The surface quantities should be greater when the maximum velocity is at the surface.

Conclusions.

It is believed that the suspension of sediment in flowing water may be attributed to three causes:

First.—The resultant upward thrust due to eddies, conditioned upon the facts that the earth profile offers more rugosities than the air profile and the effort exerted by a current upon a solid varies as the square of the relative velocity.

Second.—The resultant upward motion of solids due to the fact that an immersed body tends to move faster than the mean velocity of the

[1] "Sur la Marche des Bateaux," *Annales des Ponts et Chaussées*, January, 1886, pp. 199-242.

displaced water and in such motion tends to follow the line of least resistance.

Third.—The viscosity of the water.

All of these causes will be present in every stream flowing under natural conditions. The first two causes will alternate in efficiency with the complex motion of the stream. At certain points of the vertical curves of velocities the second cause may entirely disappear as the curve becomes irregular, but such conditions are abnormal.

Experimental research is needed for further progress along these lines. Some of the more important questions awaiting investigation will be indicated.

Can it be shown experimentally that the power of suspension in flowing water increases with the increase of relative velocities in the vertical?

Can a measurable difference in volume of displacement be detected for the same body in quiescent and in flowing water?

What is the normal form of the vertical and horizontal sediment curve?

Can it be shown by experiment that suspending power increases with depth, the mean velocity remaining unchanged?

What is the influence of a contraction in width upon the form of the vertical curves of velocity and sediment?

What is the form of the vertical curve of velocities corresponding to incipient dragging for stream beds composed of the range of materials usually found in practice?

Can it be shown by experiment[1] that a heavier load of fine than of coarse particles can be carried with the same expenditure of stream energy?

What is the effect of temperature upon viscosity and mechanical suspension?

An apparatus has been designed for the purpose of studying the first of these questions. It consists of a long, narrow trough, fitted with glass sides for nearly its whole length, which is joined to a wooden receiving reservoir. A steady flow is set up from the reservoir through the experimental channel.

Two submerged jets, each covering the entire width of the channel,

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[1] A general method of experiment has been indicated by Ashbel Welsh, Past-President Am. Soc. C. E., *Transactions*, American Society of Civil Engineers, Vol. xi, p. 162.

are introduced near the reservoir through suitable copper guides, arranged so as to create the minimum amount of agitation in the jets and in the steady flow progressing independently.

These jets discharge under a head of 40 ft. One is placed near the bed, another near the surface of the channel. The object sought in the design was to obtain a means of controlling the form of the vertical velocity curves by varying the discharge from each of the jets. This is controlled by stop-cocks. The velocities at different parts of the vertical are measured by a cluster of Pitot's tubes.

The experiments have not yet been made. Preliminary trials have shown, however, that the control of the curve of velocities is appreciable for a sufficient distance from the jets to afford ample field for experimentation.

The author takes pleasure in expressing his deep sense of obligation to Prof. E. A. Fuertes for constant assistance and suggestions, and to Prof. I. P. Church for kindly criticism, which has led to modifications in some of the views expressed. This opportunity is gladly taken to mention the kindness of M. Edouard Collignon, Inspecteur Général des Ponts et Chaussées, in affording, among other courtesies, unusual library privileges at the École Nationale des Ponts et Chaussées, Paris; and of Prof. Conrad Zschokke, of Zurich, in assistance rendered in various ways. He wishes, also, to express his appreciation of the continued courtesy shown him by M. Cordier, the librarian of the Ecole des Ponts et Chaussées at Paris, and by Prof. Rudio, the librarian of the Zurich Polytechnikum, during his use of those libraries.

DISCUSSION.

T. C. CLARKE, M. Am. Soc. C. E.—It is rather surprising to notice in the paper the statement that no measurements have ever established the fact that there may be a convexity in the center of a stream. It is the opinion of pilots on the Mississippi River, who observe such matters very closely, that a rising river can be distinguished from a falling river by the fact that on a rising river the driftwood floats along the shores, while on a falling river it floats in the center of the stream. The belief is that a rising river is convex and the driftwood floats away to the sides, and that a falling river is concave and the driftwood comes to the center. Mr. Clarke.

L. L. BUCK, M. Am. Soc. C. E.—In the rapids of the Niagara River, below the falls and above the whirlpool, the water at the center of the stream is about 10 or 12 ft. above the level at the sides, the height being greatest where the stream has the highest velocity. At one part of the whirlpool there is also an elevation of the surface of perhaps more than a foot, due to the fact that two currents at different depths pass at this place and force the water upwards. Mr. Buck.

A possible explanation of the curvature of the surface of the Mississippi River is that the slope of a river in a given distance is less during falling than during rising stages. If it is true that the slope in a given mile causes the piling up of the water in midstream, owing to the greater velocity there, the diminished slope during a falling stage of the river would account for the concave surface at such time.

FOSTER CROWELL, M. Am. Soc. C. E.—The explanation of the convexity of surface of a rising river may be that at such a time the entire section of the river is rising throughout a long reach, and the accumulation of water will naturally take the path of least resistance. Consequently, it will have an opportunity to concentrate farthest from the banks, which will exercise a retarding influence on the flow. When the river is falling, the principle of least resistance will explain the concave surface. There, of course, can be no movement of free water without a surface slope in its direction. Mr. Crowell.

During a large flood in the Chemung River the driftwood, instead of passing down the river, left the center and moved up stream at the sides; in other words, eddies were formed along the banks. The fact that the driftwood did proceed up the river was made a great deal of by some parties who were plaintiffs in suits brought to recover damages from a railroad company owning a bridge which was claimed by them to have increased their losses. Although the bridge with all the country about it was submerged, the fact that driftwood was seen to pass up the stream along the banks was offered as positive proof that it was the bridge that had dammed the whole river and turned the current up stream. In this case the driftwood must have moved along the line of least resistance, and it may be assumed, therefore, that the surface of the water in the center of the river was higher than at the sides.

CORRESPONDENCE.

M. Partiot. M. LEON PARTIOT.*—The lateral and vertical currents in a river are generally determined by the condition of the bed of the stream, which is, in turn, subject to their action. An intimate relation connects the width, slope and depth of a river with the velocity and movement of the water. It is to be hoped that the author's interesting researches will be continued so as to carry as far as possible toward completion the study of the reciprocal action of currents and river beds. New and useful information concerning the controlling laws will probably be found in this way.

A sudden increase of the width leads to a corresponding shoaling. The water which is flowing upward is then lifted by the slope on which it rises, and, if the upper currents are not swift, it reaches the surface, where its presence is indicated by a special motion. The appearance of the surface often indicates the nature of the bottom in this way during calms, and allows boatmen to shape the course they should follow.

Other facts of the same nature may be observed, such as transverse currents in the bends of a stream. The water which is directed by the upper part of the bed against the concave bank strikes that bank and rises slightly. Its head reacts on the lower liquid strata and produces, with them, a descending current. This current, in connection with that of the lower strata, attacks the concave bank, and the eroded material is transported down stream and toward the bottom. There is thus formed in the body of the water a current which follows a more or less helicoidal curve down stream.

The solid particles which it carries are deposited upon the convex bank and form a shoal below the point corresponding to the apex of the curve of the concave bank. This shoal decreases the width of the river, and thrusts the water toward the prolongation of the latter bank, so that the deep part due to the concave curve is prolonged down stream a certain distance from the apex of that curve. The mean and uniform depth suited to the width of the stream is only produced in a cross-section below this shoal.

The formula—

$$P = m\,\frac{q\,V^2}{Rg}$$

which the writer has given† to express the eroding force, P, of the water in a concave curve makes this force vary as the square of the velocity, V, of the current and inversely as the radius of curvature, R, of the bank. When a longitudinal dike is constructed along a concave bank, the friction of the water against the dike diminishes the velocity, V; in order that the eroding force, P, and, consequently, the depth along the dike, may be maintained, the radius of curvature of the dike should decrease inversely as the square of the velocity. This rule can often be applied, although not always.

* Inspector-General, Ponts et Chaussées, Paris.

† Mémoire sur les Sables de la Loire, p. 27, Paris, 1871.

M. A. FLAMANT.*—The author is to be congratulated on the remarkable manner in which he has analyzed and summarized the numerous and varied works produced on the important subject of the suspension of solids in flowing water, which, nevertheless, is far from being completely elucidated. The writer agrees entirely with the author's statement that new experiments are needed, and has little to say, except that the ninth conclusion, on page 314, is apparently expressed in terms exceeding the author's meaning. M. Flamant.

"9. The weight of suspended material per cubic unit of water decreases from rise to embouchure."

The source of rivers is generally clear and limpid, and the quantity of suspended matter increases at first.

In order to express the proposition in an exact manner it would be necessary to take into account, at each point of a stream, the state of the banks and bed, and compare the weight of the matter previously in suspension and deposited there by the water on account of a diminution in velocity with the weight of the matter eroded by the current from the banks and bed and taken in suspension at the point considered. The amount of matter in suspension per unit volume of water increases or decreases according as the second of these two weights is greater or less than the first.

The two phenomena of deposition and erosion may very well occur simultaneously at the same point or at closely neighboring points of the same stream, and the writer does not at all believe that the materials from the banks in the higher portions of a river continue their course, without interruption, to the sea.

The writer considers it also necessary to take account of the greater or less facility with which the suspended materials may be divided.

In the case of the sands of the Loire, for example, the entraining current is insufficient to divide them. At the most, it gradually reduces their dimensions by mutual friction, but does not strike one particle against another with sufficient force to break them into several pieces. It may thus be said that, so far as the entraining current is concerned, the particles are indivisible, and, consequently, the quantity of suspended material diminishes proportionately to the diminution in velocity.

But it is otherwise when the suspended or entrained material consists of clay or mud, which may be regarded as indefinitely divisible, or having a condition of division increased by the least agitation of the liquid containing it. There will then be no tendency for this material to be deposited, and if at the same time the river bed is muddy, it is not impossible that under the action of the current a part of the mud of the bottom and banks may be taken into suspension.

*Inspector-General, Ponts et Chaussées, Algiers.

M. Flamant. It is probably in differences of this nature that an explanation should be sought for the variation of opinion, mentioned on page 280, between M. Fargue and other observers.

There are, then, in all streams portions in which the proportion of suspended material is increasing, and others in which it is diminishing, according to the character, volume and weight of the suspended material, and especially the readiness with which it may be divided, and also according to the nature of the banks and bottom. If the bed cannot be washed away, and the suspended materials are indivisible by the current, it is certain that the quantity of transported material can only diminish as the velocity of the current falls, and consequently the transporting power of the water decreases.

Aside from the materials in suspension, properly speaking, that is to say, those entirely surrounded and supported by the water, it is necessary to take into account in a stream the material entrained by the current which rolls on the bottom and rests at one or more points against the adjoining particles, either moving or at rest. The distinction between material entrained in this manner and that in suspension is not always drawn clearly, and the writer considers it very difficult to establish. It may frequently happen that a grain of sand, for example, which has rolled for a time along the bottom, becomes suspended momentarily by an eddy. It may also happen that a grain of sand or a pebble wears away after rolling a certain time, and, without becoming broken, acquires dimensions sufficiently small to permit it to be taken into suspension. An increase of suspended material may thus take place in the direction of the current. This phenomenon is not hypothetical, for it occurs in several streams, especially in the Rhone, which first carries pebbles rolling on the bottom, and, afterward, towards its mouth, is charged with suspended materials formed by the wearing away of the pebbles, which decrease in size rapidly as they move down stream.

The writer thinks the author's ninth proposition should be modified in that it is too absolute.

Mr. Starling. WILLIAM STARLING, M. Am. Soc. C. E.—This excellent paper contains a nearly complete summary of the existing state of knowledge in regard to the transportation of solid matter by flowing water. The bibliographical part is extremely valuable and evidently represents the result of a vast deal of research. It is a repository of what is of real worth and originality in the literature of this interesting subject.

To those engineers who are concerned with the improvement of sedimentary rivers, the subject of this paper is of the highest importance. A few years ago the whole plan of the improvement of the Mississippi River was made to depend upon the construction to be placed upon the laws governing the suspension of the silt in the stream. Upon these laws rests the decision which must be pronounced as to

the proper means of protection of the alluvial lands from overflow—whether by levees, outlets, cut-offs or what not—and also as to the best means of securing the banks from erosion. To such engineers and to the whole profession it is therefore a great service rendered to embody in one easily accessible paper the results of the researches and experiments which have been made during the last hundred years or so, and which have heretofore been widely scattered in treatises, transactions and pamphlets, in various languages, places and periods. Mr. Starling.

The paper, however, is far more than a mere bibliography. It gives a fair summary of the contents of each of the authorities mentioned, and enables the student who wishes to go deeply into the investigation of the subject to discriminate between what will probably be of use to him and what will be comparatively valueless. The second and third parts of the paper, the "Discussion of Observed Data" and the "Discussion of Theories of Suspension," are very interesting, and the evidence is very judiciously and ably summed up.

The list of authorities might probably be enlarged a little. In Richthofen's "Führer für Forschungsreisende,"* there is an excellent chapter on the mechanical work of flowing water, which contains a good summary of the knowledge and doctrine of that subject. Though founded on the observations of others, it almost rises to the dignity of an original work. The valuable paper of Dr. W. H. Brewer, in the "Memoirs of the National Academy of Sciences" for 1884, mentioned by Richthofen and others, on the influence of salts and foreign matters in solution on suspending power, has apparently escaped the author's notice. There were extensive observations made at Fulton, Tenn., on the Mississippi River, by Mr. W. G. Powless, for almost a whole year, in 1879–80.† These contain valuable data relative to the quantities of sediment carried at different stages and under varying circumstances, and its distribution in vertical planes, as well as studies of the movement of sand waves, which rank among the most nearly complete of their kind. The observations made at Carrollton, in 1879–80, under the direction of Captain Smith S. Leach, and thoroughly discussed by him,‡ and the report of Arthur Hider, M. Am. Soc. C. E., on the Lake Providence Reach,§ are evidently known to the author, but seem hardly to have been accorded that place which is due to their merit.

The paper of B. M. Harrod, M. Am. Soc. C. E., on the protection of banks, published in the "Report of the Mississippi River Commission" for 1885,‖ contains an excellent summary of the sediment

* Pages 133–208.

† In the "Report of the Mississippi River Commission" for 1881, pp. 66–121.

‡ "Report of the Mississippi River Commission" for 1882, pp. 98–115.

§ "Report of the Mississippi River Commission" for 1882, pp. 80–98.

‖ Afterwards printed, in a modified form, as a separate pamphlet, "On the Caving Banks of the Mississippi River." New Orleans, 1886.

Mr. Starling. observations made on the Mississippi up to that date, with comments and deductions of much value.

The voluminous work of Mr. J. G. W. Fijnje, in the Dutch language, entitled "Considerations on Certain Rivers, Including Those of the Netherlands,"* contains the records of some valuable investigations made on the lower Rhine, under the auspices of the Dutch government, and a thorough discussion of them, as well as an elaborate examination of the data derived from foreign experience, with particular reference to the relation between stage of river and quantity of sediment carried, and to the proportions of the latter in the different reaches of the stream, upper and lower. There are also the record of experiments on the influence of salt water on the subsidence of suspended matter, and observations on the distribution of sediment in vertical and horizontal planes.

Some interesting and important "Observations upon the Erosion in the Hydrographic Basin of the Arkansas River,"† were published by John Casper Branner, Ph.D. (then State Geologist of Arkansas), in 1893. These observations contained, among other things, 179 determinations of matter carried in suspension, and a smaller number of determinations of matter carried in solution. They were taken at Little Rock during the year 1887–88. As the pamphlet was published in a very limited edition, some extracts from it are given:

"The matter in suspension is greatest during a sudden high rise; but after the water in the stream stands at any high mark for a few days, the decrease of the amount of suspended matter it carries is very marked. The amount of sediment carried by the river varies widely also with the same gauge-reading at any stage, being greater with a rising and less with a falling river.

"The greatest amount of mechanical sediment found in the water during the year under consideration was 225 grains to the gallon ($= \frac{1}{259}$) when the river stood at 17 ft. on the gauge, and after protracted rains (extreme high water at Little Rock is about 28 ft.). It should be added, however, that while this high water may be taken as a type of the ordinary rises, there are times when there is but little or no rise, no increase in the volume of water discharged, but a very marked increase in the amount of mechanically suspended matter. In October, 1891, occurred one of these so-called 'red rises' of the Arkansas River, and although the river was quite low—marking only 3.9 ft. on the gauge—it carried 761 grains of matter to the gallon, of which only 46 grains was matter in solution (that is, the matter in suspension was $\frac{1}{82}$, by weight).

"The matter in solution bears no constant relation to the volume of water, though in a very general way it varies inversely with the volume of the water, and ranges from 11 to 70 grains to the United States gallon. The amount carried down, in this form, from October, 1887, to September, 1888, was 6 828 350 tons. During the single month of May, 1888, 1 161 160 tons were carried out in solution. Taking the observa-

* "Beschouwingen over eenige Rivieren, waaronder Nederlandsche," 's Gravenhage 1884–1888, 3 vols. of text and 1 of appendices. See Vol. iii, pp. 876–904.

† Reprinted from the "Wilder Quarter Century Book," Ithaca, New York, 1893.

Mr. Starling.

tions for the entire year under consideration, the matter in solution is equal to about 0.31 of that in suspension. These relations, however, are not constant. In November, 1887, for example, the dissolved matter was more than six times as much as the suspended matter—while on October 13th, 1891, the suspended matter was more than thirteen times the matter in solution."

The paper of Mr. Carl Barus, to which the author has referred in a foot-note, "On the Subsidence of Fine Particles in Liquids,"* deals merely with still water, and is not concerned with the relations between suspension and velocity. Mr. Barus shows, after Maxwell, that a drop of water whose diameter (0.025 mm.) is about one-half that of a human hair would descend in air at a rate of only 2 cm. per second. Hence a drop, the dimensions of which lie near the limit of microscopic vision, say 0.00025 mm., is practically stationary, its rate being 0.002 mm. per second. Mr. Barus thinks the suspension in water, for an indefinite time, of solid particles, is attributable partly to molecular action, partly to viscosity, partly to chemical relations between the solid and the fluid. It is found that the rate of subsidence increases rapidly with the temperature, being "enormously more rapid at 100° than at 0°." The viscosity of water at 100° is only about one-sixth of its value at zero. It is thought, however, that this accounts for only a small part of the difference in suspending power.

Mr. Barus's paper contains the results of an elaborate series of experiments on the effect of precipitants. He shows that "acids, salts, alkalies, indeed foreign material in general, when added to distilled water permanently turbid with some finely comminuted insoluble solid, in quantities not too large, increase the rate of subsidence in a marked degree, in numerous cases even many hundred-fold." It has long been known that the precipitation of sediment is much more rapid in sea water than in fresh water, and the fact has an important bearing on the formation of bars at the mouths of rivers. The paper is a very interesting one, and is well worthy of perusal.

The writer will confine his remarks mostly to the lessons which have been taught by recent experience on the Mississippi.

Reference is made† to the law of decrease of mean velocity from the rise to the embouchure of rivers. It will not be disputed that the torrential part of a stream, in time of flood, has a mean velocity greater than that of the fluvial portion; but after a river has become a river, the law, if true at all, must be understood with many reservations and exceptions. The mean velocity of the Ohio, for instance, or of the upper Mississippi, is less than that of the lower Mississippi, of which they are principal tributaries. In the flood of 1881, which was a great one in the upper Mississippi, the maximum velocity at Prescott, just below St. Paul, as determined by actual observation, was about 3 ft.

* Bulletin No. 36 of the United States Geological Survey.
† On page 280 of the paper.

Mr. Starling. per second; at Winona, 80 miles below, about 3.4 ft.; at Clayton, 100 miles below Winona, about 4 ft.; at Hannibal, about 300 miles below Clayton, about 5.2 ft.; at Grafton, about 90 miles below Hannibal, about 4.6 ft.; at St. Louis, 35 miles further down, after the reception of the Missouri, 6.8 ft. In the lower Mississippi, from Cairo to Carrollton, it was probably about 5.5 ft.

This statement is very commonly made, even in works of authority, and, no doubt, it is often true; but it is often assumed, from the well-known fact that the slope of streams generally decreases, more or less regularly, from the sources to the mouth. Calculations made by means of the ordinary formulas will therefore generally give a constantly decreasing velocity. Observations on the Mississippi, however, show a great increase of the coefficients of the formulas as the mouth of the river is approached; or, in other words, as the slope grows flatter. Not only so, but the increased volume derived from tributaries and increased drainage area often compensates, or more than compensates, for the flatter slope in the lower reaches, while in the alluvial portions the shape of the cross-section, and sometimes the improved form of the trace of the stream, is conducive to greater velocity as the mouth is neared.

The topic of greatest interest in the paper is, of course, the connection between transporting power and velocity. and this important subject receives full treatment. It is thought, however, that undue consequence is attributed to the theory of Dupuit, by which suspending power is ascribed to the difference of velocity between adjacent fillets or, rather, layers.

This explanation satisfied Humphreys and Abbot, Debauve and other engineers of reputation. The wonder is that it satisfied M. Dupuit himself, who was by no means a mere theorist or formula fiend, but a man of eminently sound common sense and of much experience. The truth is that Dupuit's theory is not at all adequate when submitted to a quantitative test to explain the observed phenomena.

To take an instance from actual practice. The average quantity of sediment carried by the Mississippi River is about $\frac{1}{3000}$ by bulk of the water. The average depth of the lower river at high water may be called about 60 ft. The maximum velocity may be taken at 6 ft., the bottom velocity at 3 ft. This may be regarded as near the extreme limit. The diameter of the largest particles carried in suspension will be taken at 0.01 ft. For the purpose of the investigation, they will be considered as cubical in form.

If the place of maximum velocity in the vertical be taken at one-third of the depth from the top, there will be a variation of 3 ft. per second in velocity in 40 ft. of depth; that is, between the upper and lower sides of a cube of an edge of 0.01 ft. there will be, at the same rate, a difference of velocity of $\frac{3}{4000}$ or 0.00075 ft. per second.

Now, a solid body of a specific gravity of 2 when immersed in water will lose one-half of its weight, as the expression is, that is, it will require a force equal to the weight of a cube of water of the same size to hold it up. The pressure of running water on the bottom face of the cube is equal to the weight of a column of water whose base is the pressed surface and whose altitude is the head, due to the velocity, multiplied by a coefficient which may be taken, on the whole, as 1. Now, if a be the edge of the cube, h the height due to the velocity v, and γ the weight of a cubic foot of water, there will result the equation— Mr. Starling.

$$\gamma a^3 = \gamma a^2 h\,;$$

that is, for a cube of the specific gravity 2, the height is equal to the edge of the cube; in the present instance 0.01 ft. The velocity due to this head is 0.8 ft.

This velocity, therefore, is required to hold up the given cube. But the difference of velocity between the upper and lower sides of the cube, as seen above, is 0.00075 ft., or only 0.001 of that required.

The smallest particles suspended in the water of the Mississippi are estimated to be about $\frac{1}{3000}$ in., or say, for convenience, 0.00003 ft. in diameter. The velocity due to this height is about 0.044 ft. The difference between the velocities at the upper and lower sides of such a cube is only $\frac{1}{444444}$, or 0.00000225; that is, about $\frac{1}{20000}$ of that required.

Instead of magnitudes derived from theory, the results given by experiment, as tabulated by the author from M. Thoulet,* may be taken. To bear up a spherical particle of specific gravity 2, of a diameter of 5 mm., a velocity of 156 mm. per second was required. The difference of velocities between the upper and lower sides of the sphere (of 0.0164 ft. diameter) is 0.00123 ft. or 0.37 mm., only $\frac{1}{432}$ of that required for suspension. A particle 0.2 mm. in diameter required a vertical velocity of 23 mm. The difference in velocities for such a body is about 0.00005 ft., or 0.015 mm., only $\frac{1}{1533}$ of that required. In this case, as in the theoretical illustration, the smaller the body, the less adequate is the explanation.

It must be remembered, too, that in Dupuit's theory the difference of velocities is not all in a vertical direction. Indeed, Dupuit has left it undetermined how much of the difference is effective for suspension. It has been assumed, in the discussion, to be all effective, in order to show the inadequacy of the theory in a striking manner.

There is a far more remarkable phenomenon than the transportation of silt by running water going on every day, and that is the suspension of solid or liquid bodies in the air. A beam of light shining through a crevice in a dark room reveals millions of small particles of perceptible and measurable size floating under the influence of gentle and ap-

* See p. 312.

Mr. Starling. parently capricious currents. Probably the greater part of these minute bodies have a specific gravity 1 000 or 1 500 times greater than that of the air in which they are suspended. Flowing water is known to be actuated by a vast number of internal currents or vortices, which, as Boussinesq has shown, consume by far the greater part of the energy of the stream. The more violent of these are plainly visible in every running water-course; and in powerful rivers, they form very marked phenomena, constituting eddies and boils which are of constant recurrence and are of great vigor. The Mississippi, to the eye, appears to be little more than a succession of such whirls. In situations where excessive disturbance of the bottom is taking place, as at crevasses in levees, the surface movements are very violent and are, indeed, a faithful index of the commotion going on below. A submerged reef or wreck, though at the depth of many feet (20 or 30, perhaps more), is marked, to the acute eye of the pilot, by a "break," as he calls it, on the surface of the river.

While it must be conceded that upward vertical movements of the particles of water will be compensated, in the long run, by downward movements of equal extent, yet, as has been observed by Mr. McMath, quoted by the author, the areas and velocities of such movements may be very different. Suppose, for instance, in a given locality, an equal number of particles to be ascending and descending; but suppose the velocity of ascent to be double that of descent, and the area occupied by the particles, consequently, only half; then the total upward thrust exercised by the current would be twice the downward thrust due to the same agency.

Captain Leach has very judiciously observed* that a distinction is to be drawn between sediment permanently and intermittently in suspension. On this discrimination hinges a very important part of the question. The lighter portion of the sediment, the fine clay or loam, may be carried almost by the mere viscosity of the water. The sand may be transported only a few miles, taken from one side of the river, added to the other, without affecting the general cross-section.† At all stages, there are areas of erosion and areas of deposition; but these areas are not the same for different stages; rather, they alternate, and the area of erosion for low water often or generally becomes the area of deposition for high.

It is possible, then, that the sediment intermittently carried from bar to pool and from pool to bar may be alternately lifted by upward currents and dropped by downward. There may, therefore, very well be a predominance of upward movement at one place and of down-

* "Report of the Mississippi River Commission" for 1882, p. 105.

† See a very interesting and able paper by J. B. Johnson, M. Am. Soc. C. E., entitled "Three Problems in River Physics," published in the *Proceedings* of the American Association for the Advancement of Science, Vol. xxxiii; also separately printed at Salem, Mass., 1885.

ward movement at another. The load of sediment intermittently carried is very great, and the material, for the most part, heavy and coarse. The alternate scour and fill at the different discharge stations on the Mississippi River usually amounts to 25 000 or 30 000 sq. ft. of cross-section annually, in a total area of 200 000 or 250 000 sq. ft. Mr. Starling.

There is still a quantity permanently suspended. This is probably light silt, either brought down by the tributaries, especially the Missouri, or picked up from the banks in exchange for heavier matter brought from above and dropped. To transport this finely divided earth is a comparatively easy task. Indeed, it might almost be carried from Cairo to New Orleans in the time required for subsidence in still water. General Comstock has found that clay, put in a glass tube and hung up with water, took three weeks to subside.

There is a great variation in the testimony of very competent observers as to the relative proportions of sediment carried in suspension and rolled along the bottom. These discrepancies were to be expected, considering the great differences in places, times and conditions which have prevailed. A very remarkable difference of opinion, however, is manifested in the reports of Arthur Hider, M. Am. Soc. C. E., and Captain Smith S. Leach, in 1880. The observations of the former were conducted at Lake Providence, of the latter at Carrollton. The two stations are 415 miles apart. The latter is near the mouth of the river. Mr. Hider thought the proportion of sediment rolled to that suspended about 10 per cent. Captain Leach thought that 0.8% was a full estimate.

It may very well be that both these engineers were right. There is no reason whatever why the relative proportions of the burden borne at the two stations should be the same. The materials transported are very different. At Lake Providence they consist largely of coarse sand and gravel; at Carrollton probably, for the most part, of light sand and clay. At Fulton, Mr. Powless thought the proportion about 2.5 per cent. His estimates were made, however, from the movements of the sand reefs, and these, in Mr. Hider's opinion, constitute but a small part of the total movement, large quantities being swept along close to the bottom and not detected in sounding.

It is probable that low-water years, on the whole, are periods of deposition, and high-water years periods of scour. At least, this seems to be the conclusion to which observations on the Mississippi point. If this opinion be confirmed, it will lead to the inference that silt-bearing streams have at some times insufficient power to carry to the sea the burdens of sediment derived from their tributaries; and that at others they have a superfluous amount of energy, so as to enlarge their beds. It will be evident, then, that great caution should be used in drawing conclusions from a small number of observations, or those covering limited periods of time.

Mr. Starling. On page 300 of the paper is given a table, indicating the distribution of sediment in a vertical sense. To the figures there set forth may be added the results of Mr. Powless's observations at Fulton, Tenn., in 1879–80. This engineer found the distribution to be as follows: Surface, 27.5%; mid-depth, 36; bottom, 36.5.

The table on page 310, showing the dimensions, movement, etc., of sand waves, may also be extended a little from the same authority. At Fulton: Mean velocity, 5.8; average depth, 45—60; height of waves (average), 4.7; length, 600; movement of crest per day, 22. At Bullerton: Velocity, 3.5; depth, 10—30; height of wave, 4.4; length, 300; movement of crest, 13; observer, W. G. Powless; date, November, 1879—November, 1880; authority, Report of Chief of Engineers for 1881.

The author is entitled to the thanks of the profession for the learning, industry and ability displayed in presenting to them so much valuable information; and to those of this Society in particular for enriching its *Transactions* with so excellent a paper.

Mr. Wisner. GEORGE Y. WISNER, M. Am. Soc. C. E.—The author has placed before the Society in his interesting and valuable paper nearly all the available data relative to the transporting power of water in motion, and plainly states the hydraulic laws governing such action. In his conclusions he attributes the suspension of sediment in flowing water to three causes, viz., the resultant upward thrust due to eddies, the resultant upward motion of solids due to the fact that an immersed body tends to move faster than the mean velocity of the displaced water, and the viscosity of the water. To these three causes should also be added a fourth, due to change of curvature of the river bed, which in alluvial streams is the principal factor tending to produce unequal depths in the channel. The velocity of the current for the upper two-thirds of the depth, being much greater than that of the bottom stratum, forces the main volume of the water against the concave bank and causes a decided inclination of the surface towards the convex bank; and since this movement is very light near the river bottom, the difference of level produces a transverse pressure on the particles of the river bed, which may be many times that due to the normal current in a straight reach of the river. The molecules of water, after impinging along the concave bank, will be forced down by the pressure diagonally towards the bottom, and thence across the course of the threads of the current along the river bed, thus forming a system of converging forces, the resultants of which constitute the eroding forces which tear up particles of the bed near the concave bank and deposit them in the slack water along the opposite shore. The maximum effect of these converging forces occurs at times of high water, producing over-saturation in curves, and causing bar formation in straight reaches immediately below. At times of low water these deep pools are practically level, and the river slope is concentrated in

the short reaches across bars through which low-water channels are eroded, giving rise to the well-known anomaly of bar formation at high stage of alluvial rivers and channel deepening at low stage. Mr. Wisner.

At crossings between curves a cross-section normal to the direction of flow is usually much wider than a mean width of channel; and, the transporting power of the river consequently being less, a deposit takes place.

Aside from the effect of eddies and curves, the relative velocities of the strata at different depths create the force tending to hold matter in suspension, and since relative velocities for two streams having different depths and the same mean velocities will be less in the deeper stream, the shallow stream will have the greater eroding power.

A particle of sediment in suspension having a greater density than water will undoubtedly be subject to an accelerating force tending to move it faster than the current, equal to the specific weight of the particle in the fluid multiplied by the sine of the angle of the slope. The action of the relative velocities of successive strata and of the accelerating force due to gravity will tend to lift the particle until the plane of maximum current velocity is reached, above which the effect of the relative velocities will be negative. Consequently a greater amount of sediment should not be found at the surface than at three-tenths of the depth below the surface, as stated by Humphreys and Abbot in their Report on the Mississippi River.

A revolving ball thrown horizontally in the air, or a flat stone thrown along the surface of still water, will curve either to the right or left according to the direction of revolution. Owing to the viscosity of water the slower revolving motion of a particle in suspension, due to the unequal velocity of current on opposite sides, will for similar reasons give an upward curve to its path through the water in which it is suspended until it is lifted to the plane of maximum velocity, above which the velocities decrease towards the surface and produce a downward pressure on particles brought to the surface by eddies.

Since a certain amount of force is expended in lifting a particle from the bottom, the effect is to reduce the relative velocities of the strata through which it passes, and since the mean velocity of the stream is a function of these relative velocities, it is evident that the mean velocity of a stream for a given stage must decrease with the load carried in suspension.

For any given mean velocity of flow a shallow stream requires a much greater slope than a deep river, and as steep slopes give strong relative velocities from the bottom upwards, the transporting power of a river is much greater on the upper tributaries than near the mouth. For the same reason, the amount of sediment which a stream will transport, for any given stage when rising, is much greater than

Mr. Wisner. for the same depth when the river is falling, since the slope is always greater in front of the flood crest.

Airy and Law show theoretically that the eroding power of a stream varies as the sixth power of the bottom velocity, but it must not be inferred from this that the eroding force and transporting power vary as the sixth power of the mean velocity of the river, as has been stated in recent works on this subject. It is quite certain that the bottom velocity of a stream does not increase in the same ratio as the mean velocity, and it is probable that bottom velocities are functions of the roughness of the river bed and of the relative velocity of current from the bottom upward, or the river slope, since the relative velocities depend on slope. Airy's formula also assumes a direct impact of the current on the particle, which is seldom the case, and since the thread of the current diverted from its path by a grain of sediment on the bottom loses but little momentum, it is very probable that the actual effect is much less than the formula would indicate.

The loss of force from impact varies with the depths of the stream, and consequently a given current velocity might produce scour in a shallow stream and have no effect whatever in deeper channels. It is therefore evident that Airy's formula and Dubuat's velocities at which different sizes of gravel will commence to move in currents have but little value in practice.

Mr. Le Conte. L. J. Le Conte, M. Am. Soc. C. E.—Airy's fundamental law that the transporting power of flowing water varies as the sixth power of the velocity is as true and immutable as Torricelli's general law in hydraulics that v varies as $\sqrt{2\,g\,h}$. It is curious to note how many practical men still dispute the truth of Airy's law, but the facts which they present to substantiate their views and conclusions show but too plainly that they are laboring under a misapprehension of the first magnitude.

Nearly all the difficulty in comprehending Airy's law can be overcome entirely by simply noticing the fact that this law bears the same relation to the transporting power of water that Torricelli's law, v varies as $\sqrt{2\,g\,h}$, bears to practical engineering formulas now in use for the flow of water. It is simply a general law, and consequently is of little or no practical value at the present time, for the obvious reason that the influence of friction, viscosity, and all the other unknown disturbing causes, is yet to be introduced. When discovered and understood, these factors will some day modify the general law. In the present state of the knowledge of this subject, for any one to try a simple experiment and then proclaim Airy's law all wrong, is on a par with the procedure of a young engineer who not long ago tried an experiment with a water supply pipe line, and made the discovery that the velocity of discharge did not equal $\sqrt{2\,g\,h}$, and therefore Torricelli's law was all wrong.

It is quite clear that Airy's law will have to be supplemented by a large amount of experimental research as to modifying coefficients before it will be in such a shape as to be of much practical use. Here is an opportunity for some young ambitious member to render a great public service. When such experiments are completed, engineers will no doubt have many useful transporting-power formulas, which will certainly rival Kutter's in the number and complexity of their sub-coefficients Mr. Le Conte.

The author calls attention to the wide difference in opinion among river engineers as to the ratio of material carried in suspension and that moving along the bottom. This can all be explained if the character of the stream in each case be fully considered. The writer's experience has been largely confined to rivers similar to the Missouri, and naturally his impressions accord closely with those of the Missouri River engineers. He has the best of reasons to know that in all such sedimentary streams the larger half of the material transported during the year is not carried in suspension. During flood stages especially, the entire sandy bed of the river, from bank to bank, and to a depth of 10 to 12 ft., is moving bodily down stream like an Alpine glacier. The sand waves on top are of no importance compared with the great bed-flow of sand *en masse.* Moreover, where the flood-slope is 4 ins. per mile, the transporting power of this bed-flow is such that coarse shingle and cobblestones are transported bodily down stream, corresponding in size and weight to those transported by water in the upper river, where the flood slope is 36 ins. per mile.

The author mentions a few experiments made by the writer in 1879 giving the variation in percentage of sediment in suspension in a vertical. These limited observations were taken with a certain object in view, and at a certain stage of the river, about the middle of the front slope of the flood wave, and the writer thinks the observed maximum at mid-depth is largely due to a transient condition at this stage. It may be safely stated that as a rule the weight of sediment in suspension increases more rapidly than the depth from the surface to the bottom.

As to the distribution in a horizontal line the data collected by the Missouri and Mississippi River Commissions, both agree that the percentage in suspension is practically constant all the way across the river. As to the most potent cause of suspension, the writer thinks M. Partiot is undoubtedly correct. The writer has had rare opportunities for studying the eddying motion of particles in suspension, from the point where they first leave the bottom until they reach the surface above. This upward inclined eddy motion does not in anyway conflict with the general horizontal flow of the stream. The whole motion can be perfectly understood in all its confusing details by simply comparing it with the identical phenomena observed

Mr. Le Conte. at an ordinary camp-fire in the open plain, the smoke, wafted by a gentle breeze, rising upward in the form of an inclined eddy, expanding as it rises. This likeness in the two cases is so striking than an observer watching the natural operations of suspension could well imagine himself in a balloon looking down upon a multitude of tiny camp-fires, and a gentle uniform breeze wafting the myriads of rising columns of smoke, so that they all incline in one direction. All the complex motions in the two cases are in every respect identical. A slight irregularity in the bottom is all that is necessary to start and maintain an eddy.

The author touches lightly on the subject of viscosity. The writer has given much attention to studying the effects of viscosity on the transporting power of running water. The phenomena are fully developed in hydraulic dredging operations on river and harbor works, the column of water discharged from the large pumps being charged with all percentages and every class of material, some in suspension and some rolling along the bottom. The results show conclusively the powerful influence of viscosity due to sediment in suspension. A channel lined with cobblestones will stand a clear-water velocity of 6 to 8 ft. per second, but when it is followed by a more viscous stream heavily charged with material in suspension, although at a reduced velocity of only 3 to 4 ft. per second, the cobblestones will be at once picked up and transported with the flow.

Mr. Hooker. Elon Huntington Hooker, Ph. D.—It is to be hoped that advantage may be taken of the opportunity offered by the Niagara River to put at rest the question of surface convexity in a river at a stage of permanency. The excessive velocity in the rapids makes it a favorable place to detect the phenomenon, and at the same time renders it difficult to obtain those cross-sections of surface and bed which would show the convexity, and also the fact that it was not due to local distortions of the bed. The measurements known to the author do not warrant the statement that surface convexity exists at a permanent stage with normal form of bed.

The influence of curves in producing local suspension of sediment has been referred to in the discussion. The principle of scour at the concave bank and deposit at the convexity opposite, occurring in each case below the point of maximum curvature, must have its logical outcome in a longitudinal sinuous movement of the stream bed. M. Clavel* has been able to trace this movement clearly in the Garonne at Caudrot. The sinuousities are marked, and maps of the years 1780, 1806, 1822 and 1840 give opportunities for comparison by superposition, as shown by the plates accompanying his article.

* Annales des Ponts et Chaussées, April, 1895.